Confidence Intervals and Hypothesis Testing for TWO Populations

A sample of 25 malpractice lawsuits filed against doctors showed that the mean compensation awarded to the plaintiffs was $410,425 with a standard deviation of $74,820. Find a 95% confidence interval for the mean compensation awarded to plaintiffs of all such lawsuits. Assume that the compensations awarded to plaintiffs of all such lawsuits are normally distributed.

Independent V. Dependent Samples

- <u>Independent</u>
- Drawn from 2 different populations
- The selection of one sample does not affect the selection of the other
- Example

  Salaries of Male v Females

<u>Dependent</u>

- somehow related
- Example

  Mean weights before and after a weight loss program

With TWO samples

- Construct Confidence Interval for the *<u>difference</u>* between population means: $\mu_1 - \mu_2$
- Conduct an hypothesis test about the *<u>difference</u>* between population means: $\mu_1 - \mu_2$

- $\mu_1 \rightarrow$ mean of pop. 1
- $\mu_2 \rightarrow$ mean of pop. 2
- $\sigma_1 \rightarrow$ st. dev. of pop. 1
- $\sigma_2 \rightarrow$ st. dev. of pop 2
- $n_1 \rightarrow$ sample size drawn from pop 1
- $n_2 \rightarrow$ sample size drawn from pop 2
- $\overline{x}_1 \rightarrow$ mean of sample drawn from pop 1
- $\overline{x}_2 \rightarrow$ mean of sample drawn from pop 2

FORMULAS

- If σ_1 is <u>known</u> and the population is large or normally distributed, *and* σ_2 is <u>known</u> and the population is large or normally distributed, then

$$\mu_{\overline{x}_1 - \overline{x}_2} = \mu_1 - \mu_2 \qquad \sigma_{\overline{x}_1 - \overline{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Confidence Interval is $\left(\overline{x}_1 - \overline{x}_2\right) \pm z\sigma_{\overline{x}_1 - \overline{x}_2}$

For Hypothesis Testing, the value of the test statistic is

$$z_O = \frac{\left(\overline{x}_1 - \overline{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sigma_{\overline{x}_1 - \overline{x}_2}}$$

Constructing a Confidence Interval

<u>Ex 10-3 p. 437</u>: According to *PARADE* magazine, the average starting salaries for 2004 college grads with economics and business degrees were $40,906 and $38,188 respectively. Suppose that those averages were based on random samples of 700 econ majors and 1000 business majors with σ_1 = $5600 and σ_2 = $5900 respectively.

A) What is the point estimate of $\mu_1 - \mu_2$?

B) Construct a 97% confidence interval for $\mu_1 - \mu_2$.

Conducting an Hypothesis Test

- 700 econ majors with mean salary of $40,906 and σ_1 = $5600
- 1000 business majors with a mean salary of $38,188 and σ_2 = $5900
- Test at the 1% significance level if the population means of the starting salaries are different.

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The following information is obtained from two independent samples selected from 2 pop.

$n_1 = 300$ $\quad \overline{x}_1 = 22.0$ $\quad \sigma_1 = 4.9$

$n_2 = 250$ $\quad \overline{x}_2 = 27.6$ $\quad \sigma_2 = 4.5$

a) What is the point estimate of $\mu_1 - \mu_2$?

22 – 27.6 =-5.6

b) Construct a 95% confidence interval for $\mu_1 - \mu_2$. Find the margin of error for this estimate.

-6.39 to -4.81 with E = 0.79

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n_1 = 29 $\quad x_1$ = 101 $\quad \sigma_1$ = 15

n_2 = 27 $\quad x_2$ = 92 $\quad \sigma_2$ = 10

a) Let μ_1 and μ_2 be the population means of elapsed times for the two repellents, respectively. Find the point estimate of $\mu_1 - \mu_2$. 101 – 92 = 9 hr

b) Find a 97% confidence interval for $\mu_1 - \mu_2$

The difference is means is 1.65 to 16.35 hrs

c) Test at the 2% significance level whether the mean elapsed times for repellents A and B are different. Reject H_0 and conclude the mean times are different.