Inferences About the Difference Between Two Population Means for Paired Samples

10.4

Dependent Samples

- Mean Weight loss before and after an exercise program
- Effectiveness of fertilizer on the yield of potatoes
- Called "Paired" or "Matched" samples

- The difference between the two data values for each element of the two samples is denoted by "d"
- "Paired difference"
  - Differences are treated as one sample
- Common sample size, n
- df = n - 1

Other Notation

- $\mu_d \Rightarrow$ mean of the paired differences
- $\sigma_d \Rightarrow$ standard deviation of the paired differences of the pop.
  - Usually never known
- $\bar{d} \Rightarrow$ mean of the paired differences for the sample
- $S_d \Rightarrow$ standard deviation of the paired differences for the sample
- $n \Rightarrow$ the number of paired difference values.

If σ_d is not known and

- At least one of the following are true
  - Sample is large OR population of paired differences is normally distributed
- The t distribution is used and

$$s_{\bar{d}} = \frac{s_d}{\sqrt{n}}$$

FORMULAS

$$\bar{d} = \frac{\sum d}{n} \qquad s_d = \sqrt{\frac{\sum d^2 - \frac{\left(\sum d\right)^2}{n}}{n-1}}$$

- If σ_d is known and either the sample is large or normally distributed, $\mu_{\bar{d}} = \mu_d$ and $\sigma_{\bar{d}} = \frac{\sigma_d}{\sqrt{n}}$

Conf. Int and Hyp. Testing

- Confidence Intervals $\bar{d} \pm t s_{\bar{d}}$
- Test Statistic: $t_o = \dfrac{\bar{d} - \mu_{\bar{d}}}{s_{\bar{d}}}$

A researcher wanted to find the effect of a special diet on systolic blood pressure. The table shows the systolic blood pressure before and after 3 months of following the diet. Assume the population is normally distributed.

Before	210	180	195	220	231	199	224
After	193	186	186	223	220	183	233

Construct a 95% Confidence Interval

Before	210	180	195	220	231	199	224
After	193	186	186	223	220	183	233

Find $\bar{d}$
Calculate s_d
Calculate $s_{\bar{d}}$
Determine the value of t

- A company wanted to know if attending a course on "How to be a successful salesperson" can increase the average sales of its employees. The company sent 6 employees to attend the course.

Before	12	18	25	9	14	16
After	18	24	24	14	19	20

Before	12	18	25	9	14	16
After	18	24	24	14	19	20

- Using the 1% significance level, can you conclude that the mean weekly sales for all salespersons increase as a result of attending this course? Note: d is sales before minus sales after.