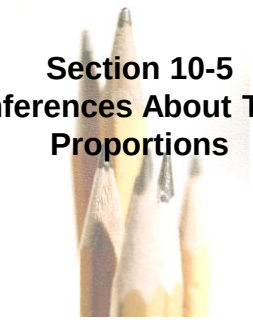
Section 10-5
Inferences About Two Proportions



Notation for Two Proportions

For population 1, we let:

p_1 = population proportion

n_1 = size of the sample

x_1 = number of successes in the sample

$\hat{p}_1 = \frac{x_1}{n_1}$ (the sample proportion)

$\hat{q}_1 = 1 - \hat{p}_1$

The corresponding notations apply to p_2, n_2, x_2, $\hat{p}_2$. and $\hat{q}_2$, which come from population 2.

Pooled Sample Proportion

- The pooled sample proportion is denoted by $\bar{p}$ and is given by:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

- We denote the complement of $\bar{p}$ by $\bar{q}$, so $\bar{q} = 1 - \bar{p}$

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Requirements

1. We have proportions from two independent simple random samples.
2. For each of the two samples, the number of successes is at least 5 and the number of failures is at least 5.

For 2 large and independent samples, the *confidence interval* for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z s_{\hat{p}_1 - \hat{p}_2}$$

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Test Statistic for Two Proportions

For $H_0: p_1 - p_2 = 0$

$H_1: p_1 - p_2 \neq 0$, $H_1: p_1 - p_2 < 0$, $H_1: p_1 - p_2 > 0$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{s_{\hat{p}_1 - \hat{p}_2}}$$

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Example :

A researcher wanted to estimate the difference between the percentages of users of 2 toothpastes who will never switch to another toothpaste. In a sample of 500 users of Toothpaste A , 100 said they will never switch. In a sample of 400 users of Toothpaste B, 68 said they will never switch.

a) What is the point estimate of $p_1 - p_2$?

b) Construct a 97% confidence interval for difference between the proportions of all users of the two toothpastes who will never switch

Example (cont):

A researcher wanted to estimate the difference between the percentages of users of 2 toothpastes who will never switch to another toothpaste. In a sample of 500 users of Toothpaste A , 100 said they will never switch. In a sample of 400 users of Toothpaste B, 68 said they will never switch. At the 1% level of significance, can we conclude that the proportion of users of toothpaste A who will never switch is higher than the proportion of users of toothpaste B who will never switch?