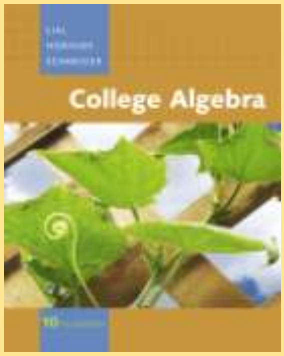
1 Equations and Inequalities

Sections 1.1–1.4



1 Equations and Inequalities

1.1 Linear Equations

Basic Terminology of Equations ▪ Solving Linear Equations ▪ Identities, Conditional Equations, and Contradictions ▪ Solving for a Specified Variable (Literal Equations)

1.1 Example 1 Solving a Linear Equation (page 85)

Solve $-4(3x-5)=3-(8x+7)$.

$$-4(3x-5)=3-(8x+7)$$

$-12x+20=3-8x-7$ Distributive property

$-12x+20=-4-8x$ Combine terms.

$24=4x$ Add 4 to both sides. Add 12*x* to both sides. Combine terms.

$6=x$ Divide both sides by 4.

Solution set: {6}

1.1 Example 2 Clearing Fractions Before Solving a Linear Equation (page 85)

Solve $\frac{3s+6}{10}-\frac{1}{2}s=\frac{2}{5}s+\frac{33}{5}$.

$$\frac{3s+6}{10}-\frac{1}{2}s=\frac{2}{5}s+\frac{33}{5}$$

$10\left(\frac{3s+6}{10}-\frac{1}{2}s\right)=10\left(\frac{2}{5}s+\frac{33}{5}\right)$ Multiply by 10, the LCD of all the fractions.

$3s+6-5s=4s+66$ Distributive property

$-2s+6=4s+66$ Combine terms.

$-6s=60$ Add –4*s* and –6 to both sides. Combine terms.

$s=-10$ Divide both sides by –6.

Solution set: {–10}

1.1 Example 3(a) Identifying Types of Equations (page 86)

Decide whether the equation is an identity, a conditional equation, or a contradiction. Give the solution set.

$$6x-9=4x+13$$

$2x=22$ Add –4*x* and 9 to both sides. Combine terms.

$x=11$ Divide both sides by 2.

This is a conditional equation.
Solution set: {11}

1.1 Example 3(b) Identifying Types of Equations (page 86)

Decide whether the equation is an identity, a conditional equation, or a contradiction. Give the solution set.

$10 + 14x = 7(2x - 5)$

$10 + 14x = 14x - 35$ Distributive property

$10 = -35$ Subtract $14x$ from both sides.

This is a contradiction.

Solution set: ø

1.1 Example 3(c) Identifying Types of Equations (page 86)

Decide whether the equation is an identity, a conditional equation, or a contradiction. Give the solution set.

$-3(2x - 1) + 5x = 3 - x$

$-6x + 3 + 5x = 3 - x$ Distributive property

$-x + 3 = 3 - x$ Combine terms.

$0 = 0$ Add x and –3 to both sides.

This is an identity.

Solution set: {all real numbers}

1.1 Example 4 Solving for a Specified Variable (page 87)

Solve for the specified variable.

(a) $d = rt$, for t

$d = rt \Rightarrow \frac{d}{r} = t$ Divide both sides by r.

(b) $S = kr^2 + kr\ell$, for k

$S = kr^2 + kr\ell$

$S = k(r^2 + r\ell)$ Factor out k.

$k = \frac{S}{r^2 + r\ell}$ Divide both sides by $r^2 + r\ell$

1.1 Example 4 Solving for a Specified Variable (cont.)

Solve for the specified variable.

(c) $11y + 8 = 2(4y + 5w) - 6z$, for y

$11y + 8 = 2(4y + 5w) - 6z$

$11y + 8 = 8y + 10w - 6z$ Distributive property

$3y = 10w - 6z - 8$ Subtract $8y$ and 8 from both sides.

$y = \frac{10w - 6z - 8}{3}$ Divide both sides by 3.

1.1 Example 5 Applying the Simple Interest Formula (page 88)

Caden borrowed \$2580 to buy new kitchen appliances for his home. He will pay off the loan in 9 months at an annual simple interest rate of 6.0%. How much interest will he pay?

$I = Prt$

$P = \$2580, t = \frac{9}{12} = \frac{3}{4}$ yr, and $r = .06$

$I = Prt = 2580(.06)\left(\frac{3}{4}\right) = \$116.10$

Caden will pay \$116.10 in interest.

1.2 Applications and Modeling with Linear Equations

Solving Applied Problems Geometry Problems Motion Problems Mixture Problems Modeling with Linear Equations

1.2 Example 1 Find the Dimensions of a Square (page 91)

The length of a rectangle is 2 in. more than the width. If the length and width are each increased by 3 in., the perimeter of the new rectangle will be 4 in. less than 8 times the width of the original rectangle. Find the dimensions of the original rectangle.

Assign variables:

Let x = the length of the original rectangle.

1.2 Example 1 Find the Dimensions of a Square (cont.)

The perimeter of the new rectangle is

The perimeter of the new rectangle is 4 in. less than 8 times the width of the original rectangle, so we have

1.2 Example 2 Solving a Motion Problem (page 92)

Krissa drove to her grandmother's house. She averaged 40 mph driving there. She was able to average 48 mph returning, and her driving time was 1 hr less. What is the distance between Krissa's house and her grandmother's house?

Let x = the distance between Krissa's house and her grandmother's house

Use $d = rt$

1.2 Example 3 Solving a Mixture Problem (page 93)

How many liters of a 25% anti-freeze solution should be added to 5 L of a 10% solution to obtain a 15% solution?

1.2 Example 3 Solving a Mixture Problem (cont.)

Create a table to show the relationships in the problem.

Strength	Gallons of Solution	Gallons of Pure Antifreeze
25%	x	$.25x$
10%	5	$.10 \cdot 5 = .5$
15%	$x+5$	$.15(x+5)$

Write an equation: $.25x + .5 = .15(x+5)$

1.2 Example 4 Solving an Investment Problem (page 94)

Last year, Owen earned a total of $1456 in interest from two investments. He invested a total of $28,000, part at 4.8% and the rest at 5.5%. How much did he invest at each rate?

Let x = amount invested at 4.8%.
Then $28,000 - x$ = amount invested at 5.5%.

1.2 **Example 4** Solving an Investment Problem (cont.)

Create a table to show the relationships in the problem.

1.2 **Example 5** Modeling the Prevention of Indoor Pollutants (page 95)

A range hood removes contaminants at a rate of *F* liters of air per second. The percent *P* of contaminants that are also removed from the surrounding air can be modeled by the linear equation

$$P = 1.06F + 7.18,$$

where $10 \le F \le 75$. What flow *F* must a range hood have to remove 70% of the contaminants from the air?

1.3 Complex Numbers

Basic Concepts of Complex Numbers ▪ Operations on Complex Numbers

1.3 **Example 1** Writing $\sqrt{-a}$ as $i\sqrt{a}$ (page 104)

Write as the product of a real number and *i*.

(a) $\sqrt{-81} = i\sqrt{81} = 9i$

(b) $\sqrt{-55} = i\sqrt{55}$

(c) $\sqrt{-98} = i\sqrt{98} = i\sqrt{49 \cdot 2} = 7i\sqrt{2}$

1.3 **Example 2** Finding Products and Quotients Involving Negative Radicands (page 105)

Multiply or divide. Simplify each answer.

(a) $\sqrt{-21} \cdot \sqrt{-21} = i\sqrt{21} \cdot i\sqrt{21} = i^2 \cdot \left(\sqrt{21}\right)^2$
$= -1 \cdot 21 = -21$

(b) $\sqrt{-5} \cdot \sqrt{-30} = i\sqrt{5} \cdot i\sqrt{30} = i^2\sqrt{150}$
$= i^2\sqrt{25 \cdot 6} = -5\sqrt{6}$

1.3 **Example 2** Finding Products and Quotients Involving Negative Radicands (cont.)

Multiply or divide. Simplify each answer.

(c) $\dfrac{\sqrt{-42}}{\sqrt{-3}} = \dfrac{i\sqrt{42}}{i\sqrt{3}} = \sqrt{\dfrac{42}{3}} = \sqrt{14}$

(d) $\dfrac{\sqrt{-63}}{\sqrt{21}} = \dfrac{i\sqrt{63}}{\sqrt{21}} = i\sqrt{\dfrac{63}{21}} = i\sqrt{3}$

1.3 **Example 3** Simplifying a Quotient Involving a Negative Radicand (page 105)

Write $\frac{15-\sqrt{-75}}{5}$ in standard form $a + bi$.

$$\frac{15-\sqrt{-75}}{5} = \frac{15-i\sqrt{75}}{5} \qquad \sqrt{-75} = i\sqrt{75}$$

$$= \frac{15-i\sqrt{25\cdot 3}}{5}$$

$$= \frac{15-5i\sqrt{3}}{5}$$

$$= \frac{5\left(3-i\sqrt{3}\right)}{5} \qquad \text{Factor.}$$

$$= 3-i\sqrt{3}$$

1.3 **Example 4** Adding and Subtracting Complex Numbers (page 106)

Find each sum or difference.

(a) $(4 - 5i) + (-5 + 8i) = [4 + (-5)] + (-5i + 8i)$
$= -1 + 3i$

(b) $(-6 + 3i) + (12 - 9i) = 6 - 6i$

(c) $(-10 + 7i) - (5 - 3i) = (-10 - 5) + [7i + (3i)]$
$= -15 + 10i$

(d) $(15 - 8i) - (-10 + 4i) = 25 - 12i$

1.3 **Example 5** Multiplying Complex Numbers (page 107)

Find each product.

(a) $(5 + 3i)(2 - 7i) = 5(2) + (5)(-7i) + (3i)(2) + (3i)(-7i)$
$= 10 - 35i + 6i - 21i^2$
$= 10 - 29i - 21(-1)$
$= 31 - 29i$

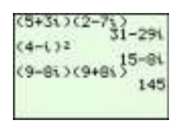
(b) $(4 - 5i)^2 = 4^2 - 2(4)(5i) + (5i)^2$
$= 16 - 40i + 25i^2$
$= 16 - 40i + 25(-1)$
$= -9 - 40i$

1.3 **Example 5** Multiplying Complex Numbers (cont.)

Find the product.

(c) $(9 - 8i)(9 + 8i) = 9^2 - (8i)^2$
$= 81 - 64i^2$
$= 81 - 64(-1)$
$= 81 + 64$
$= 145$ or $145 + 0i$

This screen shows how the TI–83/84 Plus displays the results in this example.



1.3 **Example 6** Simplifying Powers of *i* (page 107)

Simplify each power of *i*.

(a) i^{33} (b) i^{-14}

Write the given power as a product involving $i^2 = -1$ or $i^4 = 1$.

(a) $i^{33} = i^{32}\cdot i$
$= \left(i^4\right)^8 \cdot i$
$= 1^8 \cdot i$
$= i$

(b) $i^{-14} = i^{-16}\cdot i^2$
$= \left(i^{16}\right)^{-1} \cdot i^2$
$= \left(\left(i^4\right)^4\right)^{-1} \cdot i^2$
$= 1^{-1}\cdot(-1) = -1$

1.3 **Example 7(a)** Dividing Complex Numbers (page 108)

Write in standard form $a + bi$.

$$\frac{5-5i}{3+i} = \frac{5-5i}{3+i}\cdot\frac{3-i}{3-i} \qquad \text{Multiply the numerator and denominator by the complex conjugate of the denominator.}$$

$$= \frac{15-5i-15i+5i^2}{9-i^2} \qquad \text{Multiply.}$$

$$= \frac{15-20i-5}{9-(-1)} \qquad i^2 = -1$$

$$= \frac{10-20i}{10} \qquad \text{Combine terms.}$$

$$= 1-2i \qquad \text{Lowest terms; standard form}$$

1.3 **Example 7(b)** Dividing Complex Numbers (page 108)

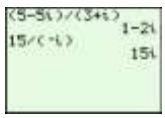
Write in standard form *a* + *bi*.

$$\frac{15}{-i} = \frac{15}{-i} \cdot \frac{i}{i}$$ Multiply the numerator and denominator by the complex conjugate of the denominator.

$$= \frac{15i}{-i^2} = \frac{15i}{1}$$ Multiply. $-i^2 = 1$

$$= 15i \text{ or } 0 + 15i$$ Lowest terms; standard form

This screen shows how the TI–83/84 Plus displays the results in this example.



1.4 Quadratic Equations

Solving a Quadratic Equation   Completing the Square
The Quadratic Formula   Solving for a Specified Variable
The Discriminant

1.4 **Example 1** Using the Zero-Factor Property (page 111)

Solve $10x^2 + x - 2 = 0$.

$$10x^2 + x - 2 = 0$$

$$(2x+1)(5x-2) = 0$$ Factor.

$$2x + 1 = 0 \quad \text{or} \quad 5x - 2 = 0$$ Set each factor equal to 0 and then solve for *x*.

$$2x = -1 \quad \text{or} \quad 5x = 2$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = \frac{2}{5}$$

1.4 **Example 1** Using the Zero-Factor Property (cont.)

Now check.

$$10x^2 + x - 2 = 0$$

$$10\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 2 \stackrel{?}{=} 0 \qquad 10\left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right) - 2 \stackrel{?}{=} 0$$

$$10\left(\frac{1}{4}\right) + \left(-\frac{1}{2}\right) - 2 \stackrel{?}{=} 0 \qquad 10\left(\frac{4}{25}\right) + \left(\frac{2}{5}\right) - 2 \stackrel{?}{=} 0$$

$$2 = 2 \checkmark \qquad 2 = 2 \checkmark$$

Solution set: $\left\{-\frac{1}{2}, \frac{2}{5}\right\}$

1.4 **Example 2** Using the Square Root Property (page 112)

Solve each quadratic equation.

(a) $x^2 = 29 \Rightarrow x = \pm\sqrt{29}$

(b) $x^2 = -144 \Rightarrow x = \pm 12i$ $\sqrt{-144} = \sqrt{-1} \cdot \sqrt{144} = \pm 12i$

(c) $(x-8)^2 = 24$

$$x - 8 = \pm\sqrt{24}$$ Generalized square root property

$$x - 8 = \pm 2\sqrt{6}$$ $\sqrt{24} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$

$$x = 8 \pm 2\sqrt{6}$$

1.4 **Example 3** Using the Method of Completing the Square, *a* = 1 (page 113)

Solve $x^2 + 10x - 20 = 0$ by completing the square.

Rewrite the equation so that the constant is alone on one side of the equation.

$$x^2 + 10x = 20$$

Square half the coefficient of x, and add this square to both sides of the equation.

$$x^2 + 10x + \left(\tfrac{1}{2} \cdot 10\right)^2 = 20 + \left(\tfrac{1}{2} \cdot 10\right)^2$$

$$x^2 + 10x + 25 = 20 + 25$$

1.4 Example 3 Using the Method of Completing the Square, $a = 1$ (cont.)

Factor the resulting trinomial as a perfect square and combine terms on the other side.

$$(x+5)^2 = 45$$

Use the square root property to complete the solution.

$$x+5 = \pm\sqrt{45}$$
$$x = -5 \pm \sqrt{45} = -5 \pm 3\sqrt{5}$$

Solution set: $\left\{-5 \pm 3\sqrt{5}\right\}$

1.4 Example 4 Using the Method of Completing the Square, $a \neq 1$ (page 113)

Solve $4x^2 + 6x + 5 = 0$ by completing the square.

Divide both sides of the equation by ***a***, *4.*

$$x^2 + \tfrac{6}{4}x + \tfrac{5}{4} = 0$$

Rewrite the equation so that the constant is alone on one side of the equation.

$$x^2 + \tfrac{6}{4}x = -\tfrac{5}{4}$$

Square half the coefficient of ***x***, *and add this square to both sides of the equation.*

$$x^2 + \tfrac{6}{4}x + \left(\tfrac{1}{2}\cdot\tfrac{6}{4}\right)^2 = -\tfrac{5}{4} + \left(\tfrac{1}{2}\cdot\tfrac{6}{4}\right)^2$$
$$x^2 + \tfrac{3}{2}x + \tfrac{9}{16} = -\tfrac{5}{4} + \tfrac{9}{16}$$

1.4 Example 4 Using the Method of Completing the Square, $a \neq 1$ (cont.)

Factor the resulting trinomial as a perfect square and combine terms on the other side.

$$\left(x + \tfrac{3}{4}\right)^2 = -\tfrac{11}{16}$$

Use the square root property to complete the solution.

$$\left(x + \tfrac{3}{4}\right)^2 = -\tfrac{11}{16}$$
$$x + \tfrac{3}{4} = \pm i\sqrt{\tfrac{11}{16}}$$
$$x = -\tfrac{3}{4} \pm i\sqrt{\tfrac{11}{16}} = -\tfrac{3}{4} \pm \tfrac{\sqrt{11}}{4}i$$

Solution set: $\left\{-\tfrac{3}{4} \pm \tfrac{\sqrt{11}}{4}i\right\}$

1.4 Example 5 Using the Quadratic Formula (Real Solutions) (page 115)

Solve $x^2 + 6x = 3$.

Write the equation in standard form.

$$x^2 + 6x - 3 = 0$$

$a = 1$, $b = 6$, $c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-3)}}{2(1)}$$

1.4 Example 5 Using the Quadratic Formula (Real Solutions) (cont.)

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{36+12}}{2} = \frac{-6 \pm \sqrt{48}}{2}$$
$$= \frac{-6 \pm 4\sqrt{3}}{2} \qquad \sqrt{48} = \sqrt{16}\cdot\sqrt{3} = 4\sqrt{3}$$
$$= -3 \pm 2\sqrt{3}$$

Solution set: $\left\{-3 \pm 2\sqrt{3}\right\}$

1.4 Example 6 Using the Quadratic Formula (Nonreal Complex Solutions) (page 115)

Solve $4x^2 = 3x - 5$.

Write the equation in standard form.

$$4x^2 - 3x + 5 = 0$$

$a = 4$, $b = -3$, $c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(5)}}{2(4)}$$

1.4 Example 6 Using the Quadratic Formula (Nonreal Complex Solutions) (page 115)

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(5)}}{2(4)}$$

$$= \frac{3 \pm \sqrt{9-80}}{8} = \frac{3 \pm \sqrt{-71}}{8}$$

$$= \frac{3 \pm i\sqrt{71}}{8}$$

$$= \frac{3}{8} \pm \frac{\sqrt{71}}{8} i$$

Solution set: $\left\{\frac{3}{8} \pm \frac{\sqrt{71}}{8} i\right\}$

1.4 Example 7 Solving a Cubic Equation (page 116)

Solve $x^3 - 125 = 0$.

$(x-5)(x^2+5x+25) = 0$ — Factor as a difference of cubes.

$x - 5 = 0$ or $x^2 + 5x + 25 = 0$ — Zero-factor property

$x = 5$ $\quad x = \frac{-5 \pm \sqrt{5^2 - 4(1)(25)}}{2(1)}$ — Quadratic formula with $a = 1$, $b = 5$, $c = 25$

$$= \frac{-5 \pm \sqrt{25-100}}{2} = \frac{-5 \pm \sqrt{-75}}{2}$$

$$= \frac{-5 \pm 5i\sqrt{3}}{2} = \frac{-5}{2} \pm \frac{5\sqrt{3}}{2} i$$

Solution set: $\left\{5, \frac{-5}{2} \pm \frac{5\sqrt{3}}{2} i\right\}$

1.4 Example 8(a) Solving for a Quadratic Variable in a Formula (page 116)

Solve $V = \frac{1}{3}\pi r^2 h$ for *r*. Use when taking square roots.

Goal: Isolate *r*.

$V = \frac{1}{3}\pi r^2 h$

$3V = \pi r^2 h$ — Multiply by 3.

$\frac{3V}{\pi h} = r^2$ — Divide by πh.

$r = \pm\sqrt{\frac{3V}{\pi h}}$ — Square root property

$r = \pm\sqrt{\frac{3V}{\pi h}} \cdot \frac{\sqrt{\pi h}}{\sqrt{\pi h}}$ — Rationalize the denominator.

$r = \frac{\pm\sqrt{3V\pi h}}{\pi h}$ — Simplify.

1.4 Example 8(b) Solving for a Quadratic Variable in a Formula (page 116)

Solve $2my^2 - ny = 3p \; (m \neq 0)$ for *y*. Use when taking square roots.

$2my^2 - ny - 3p = 0$ — Write in standard form.

$y = \frac{-(-n) \pm \sqrt{(-n)^2 - 4(2m)(-3p)}}{2(2m)}$ — Use the quadratic formula with $a = 2m$, $b = -n$, $c = -3p$.

$y = \frac{n \pm \sqrt{n^2 + 24mp}}{4m}$ — Simplify.

1.4 Example 9(a) Using the Discriminant (page 118)

Determine the number of distinct solutions, and tell whether they are *rational*, *irrational*, or *nonreal complex* numbers.

$$4x^2 - 12x + 9 = 0$$

$a = 4$, $b = -12$, $c = 9$

$$b^2 - 4ac = (-12)^2 - 4(4)(9) = 0$$

There is one distinct rational solution.

1.4 Example 9(b) Using the Discriminant (page 118)

Determine the number of distinct solutions, and tell whether they are *rational*, *irrational*, or *nonreal complex* numbers.

$$3x^2 + x = -5$$

$3x^2 + x + 5 = 0$ — Write in standard form.

$a = 3$, $b = 1$, $c = 5$

$$b^2 - 4ac = 1^2 - 4(3)(5) = -59$$

There are two distinct nonreal complex solutions.

1.4 **Example 9(c)** Using the Discriminant (page 118)

Determine the number of distinct solutions, and tell whether they are *rational*, *irrational*, or *nonreal complex* numbers.

$$2x^2 = 6x + 7$$

$$2x^2 - 6x - 7 = 0$$ Write in standard form.

$a = 2$, $b = -6$, $c = -7$

$$b^2 - 4ac = (-6)^2 - 4(2)(-7) = 92$$

There are two distinct irrational solutions.