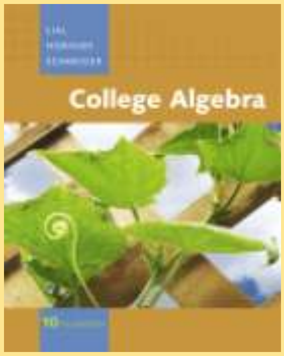
1

Equations and Inequalities

Sections 1.5–1.8



1 Equations and Inequalities

1.5 Applications and Modeling with Quadratic Equations

1.6 Other Types of Equations and Applications

1.7 Inequalities

1.8 Absolute Value Equations and Inequalities

1.5 Applications and Modeling with Quadratic Equations

Sets of Numbers and the Number Line Exponents Order of Operations Properties of Real Numbers Order on the Number Line Absolute Value

1.5 **Example 1** Solving a Problem Involving the Volume of a Box (page 122)

A box with volume 7500 cm^3 is to be formed from a sheet of metal whose length is twice the width. Equal size squares measuring 10 cm on a side are to be cut from the corners of the metal sheet in order to form the box. What are the dimensions of the original piece of metal?

Assign variables:

Let x = the width of the original rectangle.

Then, $2x$ = the length of the original rectangle.

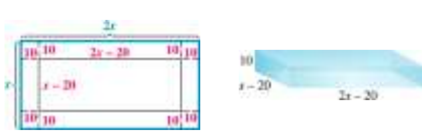
1.5 **Example 1** Solving a Problem Involving the Volume of a Box (cont.)

The box is formed by cutting 10 cm + 10 cm from both the length and the width.

$x - 20$ = the width of the bottom of the box

$2x - 20$ = the length of the bottom of the box

10 = the height of the box



1.5 **Example 1** Solving a Problem Involving the Volume of a Box (cont.)

$V = lwh = 7500$ cm^3

$$7500 = (2x - 20)(x - 20)(10)$$

$$7500 = 20x^2 - 600x + 4000 \quad \text{Multiply.}$$

$$0 = 20x^2 - 600x - 3500 \quad \text{Subtract 7500.}$$

$$0 = x^2 - 30x - 175 \quad \text{Divide by 20.}$$

$$0 = (x + 5)(x - 35) \quad \text{Factor.}$$

$$x + 5 = 0 \quad \text{or} \quad x - 35 = 0 \quad \text{Zero-factor property}$$

$$x = -5 \quad \text{or} \quad x = 35 \quad \text{Solve.}$$

Length cannot be negative. Reject $x = -5$

The dimensions of the metal piece are 35 cm x 70 cm.

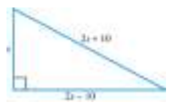
1.5 Example 2 Solving a Problem Involving the Pythagorean Theorem (page 123)

A piece of property is in the shape of the right triangle. The longer leg is 10 m shorter than twice the length of the shorter leg, and the hypotenuse is 20 m longer than the longer leg. Find the lengths of the sides of the property.

Let s = the length of the shorter leg.

Then, $2s – 10$ = the length of longer leg.

$(2s – 10) + 20 = 2s + 10$ = the length of the hypotenuse



1.5 Example 2 Solving a Problem Involving the Pythagorean Theorem (cont.)

Use the Pythagorean theorem.

$s^2 + (2s-10)^2 = (2s+10)^2$	$a^2 + b^2 = c^2$
$s^2 + 4s^2 - 40s + 100 = 4s^2 + 40s + 100$	Square the binomials.
$5s^2 - 40s + 100 = 4s^2 + 40s + 100$	Combine terms.
$s^2 - 80s = 0$	Standard form
$s(s-80) = 0$	Factor.
$s = 0$ or $s - 80 = 0$	Zero-factor property
$s = 80$	Solve.

1.5 Example 2 Solving a Problem Involving the Pythagorean Theorem (cont.)

Since s represents length, 0 is not a reasonable answer.

The lengths of the sides of the property are 80 m, 2(80) – 10 = 150 m, and 2(80) + 10 = 170 m.

1.5 Example 3(a) Solving a Problem Involving the Height of a Projectile (page 124)

If a projectile is shot upward from the ground with an initial velocity of 73.5 m per sec, neglecting air resistance, its height (in meters) above the ground t seconds after the projection is given by

$$s = -4.9t^2 + 73.5t$$

After how many seconds will the projectile be 100 m above the ground?

1.5 Example 3(a) Solving a Problem Involving the Height of a Projectile (cont.)

Find the value(s) of ***t*** *so that the height* ***s*** *is 100.*

$$s = -4.9t^2 + 73.5t$$

$$100 = -4.9t^2 + 73.5t \Rightarrow 4.9t^2 - 73.5t + 100 = 0$$

Use the quadratic formula to solve for ***t***.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 4.9,\ b = -73.5,\ c = 100$$

$$t = \frac{-(-73.5) \pm \sqrt{(-73.5)^2 - 4(4.9)(100)}}{2(4.9)}$$

$$= \frac{73.5 \pm \sqrt{3442.25}}{9.8} \approx 1.51 \text{ or } 13.49$$

1.5 Example 3(a) Solving a Problem Involving the Height of a Projectile (cont.)

Both solutions are acceptable since the projectile reaches 100 m twice, once as it rises and once as it falls.

The projectile will be 100 m above the ground after 1.51 seconds and after 13.49 seconds.

1.5 Example 3(b) Solving a Problem Involving the Height of a Projectile (page 124)

How long will it take for the projectile to return to the ground?

The projectile returns to the ground when **s** *= 0.*

$$0 = -4.9t^2 + 73.5t$$
$$0 = t(-4.9t + 73.5)$$
$$t = 0 \text{ or } -4.9t + 73.5 = 0$$
$$t = 15$$

t = 0 represents the start time.

It takes 15 seconds for the projectile to return to the ground.

1.5 Example 4(a) Analyzing Sport Utility Vehicle (SUV) Sales (page 126)

Based on figures from 1990–2001, the equation

$$S = .016x^2 + .124x + .787$$

models sales of SUVs from 1990 to 2001,where *S* represents sales in millions, and *x* = 0 represents 1990, *x* = 1 represents 1991, etc.

Use the model to determine sales in 2000 and 2001. Compare the results to the actual figures of 3.6 million and 3.7 million.

1.5 Example 4(a) Analyzing Sport Utility Vehicle (SUV) Sales (cont.)

$$S = .016x^2 + .124x + .787$$

For 2000, *x* = 10.

$$S = .016(10)^2 + .124(10) + .787 \approx 3.6 \text{ million}$$

For 2001, *x* = 11.

$$S = .016(11)^2 + .124(11) + .787 \approx 4.1 \text{ million}$$

For 2000, the prediction is equal to the actual figure of 3.6 million.

For 2001, the prediction is greater than the actual figure of 3.7 million.

1.5 Example 4(b) Analyzing Sport Utility Vehicle (SUV) Sales (page 126)

According to the model, in what year did sales reach 3 million? (Round down to the nearest year.)

$$S = .016x^2 + .124x + .787$$

Let *S* = 0, then solve for *x.*

$$3 = .016x^2 + .124x + .787 \Rightarrow 0 = .016x^2 + .124x - 2.213$$

$$x = \frac{-.124 \pm \sqrt{.124^2 - 4(.016)(-2.213)}}{2(.016)} \quad \text{Quadratic formula}$$

$$x \approx 8.5 \text{ or } x \approx -16.3$$

1.5 Example 4(b) Analyzing Sport Utility Vehicle (SUV) Sales (cont.)

Reject the negative solution, and round 8.5 down to 8.

The year 1998 corresponds to *x* = 8.

SUV sales reached 3 million in 1998.

1.6 Other Types of Equations and Applications

Rational Equations Work Rate Problems Equations with Radicals Equations Quadratic in Form

1.6 Example 1(a) Solving Rational Equations that Lead to Linear Equations (page 133)

Solve $\frac{2x-3}{2}+\frac{5x}{x+1}=x$.

The least common denominator is $2(x + 1)$, which equals 0 when $x = −1$. Therefore, −1 cannot be a solution of the equation.

1.6 Example 1(a) Solving Rational Equations that Lead to Linear Equations (cont.)

$$\frac{2x-3}{2}+\frac{5x}{x+1}=x$$

$$2(x+1)\left(\frac{2x-3}{2}+\frac{5x}{x+1}\right)=2(x+1)x$$ Multiply by the LCD, $2(x + 1)$.

$$(x+1)(2x-3)+2(5x)=2x^2+2x$$ Simplify.

$$2x^2-3x+2x-3+10x=2x^2+2x$$ Multiply.

$$2x^2+9x-3=2x^2+2x$$ Combine terms.

$$7x=3 \Rightarrow x=\tfrac{3}{7}$$

The restriction, $x \neq −1$, does not affect this result.

1.6 Example 1(a) Solving Rational Equations that Lead to Linear Equations (cont.)

Now check.

$$\frac{2x-3}{2}+\frac{5x}{x+1}=x$$

$$\frac{2\left(\frac{3}{7}\right)-3}{2}+\frac{5\left(\frac{3}{7}\right)}{\left(\frac{3}{7}\right)+1}\stackrel{?}{=}\left(\tfrac{3}{7}\right)$$

$$\frac{3}{7}=\frac{3}{7} \checkmark$$

Solution set: $\left\{\frac{3}{7}\right\}$

1.6 Example 1(b) Solving Rational Equations that Lead to Linear Equations (page 133)

Solve $\frac{x}{x-5}+5=\frac{5}{x-5}$.

The least common denominator is $x − 5$, which equals 0 when $x = 5$. Therefore, 5 cannot be a solution of the equation.

1.6 Example 1(b) Solving Rational Equations that Lead to Linear Equations (cont.)

$$\frac{x}{x-5}+5=\frac{5}{x-5}$$

$$(x-5)\left(\frac{x}{x-5}+5\right)=(x-5)\left(\frac{5}{x-5}\right)$$ Multiply by the LCD, $x – 5$.

$$x+5(x-5)=5$$ Simplify.

$$6x-25=5 \Rightarrow 6x=30 \Rightarrow x=5$$

The only possible solution is 5. However, the variable is restricted to real numbers except 5.

Solution set: Ø

1.6 Example 2(a) Solving Rational Equations that Lead to Quadratic Equations (page 134)

Solve $\frac{x-5}{x-3}+\frac{1}{x}=\frac{-7}{x^2-3x}$.

The least common denominator is $x(x-3)=x^2-3x$, which equals 0 when $x = 0$ or $x = 3$. Therefore, 0 and 3 cannot be solutions of the equation.

1.6 Example 2(a) Solving Rational Equations that Lead to Quadratic Equations (cont.)

$$\frac{x-5}{x-3}+\frac{1}{x}=\frac{-7}{x^2-3x}$$

$$x(x-3)\left(\frac{x-5}{x-3}+\frac{1}{x}\right)=x(x-3)\left(\frac{-7}{x^2-3x}\right)$$ Multiply by the LCD, $x(x-3)$.

$$x(x-5)+(x-3)=-7$$
$$x^2-5x+x-3=-7$$ Distributive property

$$x^2-4x+4=0$$ Standard form

$$(x-2)(x-2)=0$$ Factor.

$$x-2=0 \Rightarrow x=2$$

The restrictions $x \neq 0$ and $x \neq 3$ do not affect the result.

1.6 Example 2(a) Solving Rational Equations that Lead to Quadratic Equations (cont.)

Now check.

$$\frac{x-5}{x-3}+\frac{1}{x}=\frac{-7}{x^2-3x}$$
$$\frac{2-5}{2-3}+\frac{1}{2}\stackrel{?}{=}\frac{-7}{2^2-3(2)}$$
$$\frac{-3}{-1}+\frac{1}{2}\stackrel{?}{=}\frac{-7}{-2}$$
$$\frac{7}{2}=\frac{7}{2}\ ✓$$

Solution set: {2}

1.6 Example 2(b) Solving Rational Equations that Lead to Quadratic Equations (page 134)

Solve $\frac{x}{x+5}+\frac{5}{x-5}=\frac{50}{x^2-25}$.

The least common denominator is $(x+5)(x-5)=x^2-25$, which equals 0 when $x =$ 5. Therefore,   5 cannot be solutions of the equation.

1.6 Example 2(b) Solving Rational Equations that Lead to Quadratic Equations (cont.)

$$\frac{x}{x+5}+\frac{5}{x-5}=\frac{50}{x^2-25}$$
$$(x+5)(x-5)\left(\frac{x}{x+5}+\frac{5}{x-5}\right)=(x+5)(x-5)\left(\frac{50}{x^2-25}\right)$$
$$x(x-5)+5(x+5)=50$$
$$x^2-5x+5x+25=50$$
$$x^2=25 \Rightarrow x=\pm 5$$

The possible solutions are   5. However, the variable is restricted to real numbers except   5.

Solution set: Ø

1.6 Example 3 Solving a Work Rate Problem (page 135)

Lisa and Keith are raking the leaves in their backyard. Working alone, Lisa can rake the leaves in 5 hr, while Keith can rake them in 4 hr. How long would it take them to rake the leaves working together?

Assign variables.

Let x = the amount of time it would take them to rake the leaves together.

In 1 hr, Lisa can do $\frac{1}{5}$ of the job by herself.

In 1 hr, Keith can do $\frac{1}{4}$ of the job by himself.

1.6 Example 3 Solving a Work Rate Problem (cont.)

Write an equation.

	r	t	Part of the Job Accomplished
Lisa	$\frac{1}{5}$	x	$\frac{x}{5}$
Keith	$\frac{1}{4}$	x	$\frac{x}{4}$

$A = rt$

part of the job by Lisa + part of the job by Keith = One whole job

$$\frac{x}{5}+\frac{x}{4}=1$$

1.6 Example 3 Solving a Work Rate Problem (cont.)

$$\frac{x}{5}+\frac{x}{4}=1$$

$$20\left(\frac{x}{5}+\frac{x}{4}\right)=20\cdot 1$$ Multiply by the LCD, 20.

$$4x+5x=20$$ Distributive property

$$9x=20 \Rightarrow x=\frac{20}{9}=2\frac{2}{9}$$

It will take Lisa and Keith $2\frac{2}{9}$ hr working together to rake the leaves.

1.6 Example 4 Solving an Equation Containing a Radical (Square Root) (page 138)

Solve $x-\sqrt{4x+12}=0$.

$$x-\sqrt{4x+12}=0$$

$$x=\sqrt{4x+12}$$ Isolate the radical.

$$x^2=4x+12$$ Square both sides.

$$x^2-4x-12=0$$ Solve the quadratic equation.

$$(x-6)(x+2)=0$$ Factor.

$$x-6=0 \text{ or } x+2=0$$ Zero-factor property

$$x=6 \text{ or } x=-2$$ Proposed solutions

1.6 Example 4 Solving an Equation Containing a Radical (Square Root) (cont.)

Now check.

$$x-\sqrt{4x+12}=0$$

$$-2-\sqrt{4(-2)+12}\overset{?}{=}0$$
$$-2-\sqrt{4}\overset{?}{=}0$$
$$-2-2\overset{?}{=}0$$
$$-4\neq 0$$

–2 is not a solution.

$$6-\sqrt{4(6)+12}\overset{?}{=}0$$
$$6-\sqrt{36}\overset{?}{=}0$$
$$6-6\overset{?}{=}0$$
$$0=0 \checkmark$$

Solution set: {6}

1.6 Example 5 Solving an Equation Containing Two Radicals (page 138)

Solve $\sqrt{3x+1}-\sqrt{x+4}=1$.

$$\sqrt{3x+1}-\sqrt{x+4}=1$$

$$\sqrt{3x+1}=\sqrt{x+4}+1$$ Isolate one radical.

$$\left(\sqrt{3x+1}\right)^2=\left(\sqrt{x+4}+1\right)^2$$ Square both sides.

$$3x+1=\left(\sqrt{x+4}\right)^2+2\sqrt{x+4}+1$$ Expand.

$$3x+1=x+4+2\sqrt{x+4}+1$$

$$2x-4=2\sqrt{x+4}$$ Combine terms and isolate the radical.

1.6 Example 5 Solving an Equation Containing Two Radicals (cont.)

$$2x-4=2\sqrt{x+4}$$

$$(2x-4)^2=\left(2\sqrt{x+4}\right)^2$$ Square both sides.

$$4x^2-16x+16=4x+16$$ Expand.

$$4x^2-20x=0$$ Combine terms, then solve the quadratic equation.

$$x^2-5x=0$$

$$x(x-5)=0$$

$$x=0 \text{ or } x=5$$ Possible solutions

1.6 Example 5 Solving an Equation Containing Two Radicals (cont.)

Now check.

$$\sqrt{3x+1}-\sqrt{x+4}=1$$

$$\sqrt{3(0)+1}-\sqrt{(0)+4}\overset{?}{=}1$$
$$1-2\overset{?}{=}1$$
$$-1\neq 1$$

0 is not a solution.

$$\sqrt{3(5)+1}-\sqrt{(5)+4}\overset{?}{=}1$$
$$4-3\overset{?}{=}1$$
$$1=1 \checkmark$$

Solution set: {5}

1.6 Example 6 Solving an Equation Containing a Radical (Cube Root) (page 139)

Solve $\sqrt[3]{5x^2-12x+6}-\sqrt[3]{x}=0$.

$\sqrt[3]{5x^2-12x+6}-\sqrt[3]{x}=0$

$\sqrt[3]{5x^2-12x+6}=\sqrt[3]{x}$ — Isolate one radical.

$\left(\sqrt[3]{5x^2-12x+6}\right)^3=\left(\sqrt[3]{x}\right)^3$ — Cube both sides.

$5x^2-12x+6=x$ — Expand.

$5x^2-13x+6=0$ — Standard form

$(5x-3)(x-2)=0$ — Factor.

$5x-3=0$ or $x-2=0$ — Zero-factor property

$x=\frac{3}{5}$ or $x=2$ — Proposed solutions

1.6 Example 6 Solving an Equation Containing a Radical (Cube Root) (cont.)

Now check.

$\sqrt[3]{5x^2-12x+6}-\sqrt[3]{x}=0$

$\sqrt[3]{5\left(\frac{3}{5}\right)^2-12\left(\frac{3}{5}\right)+6}-\sqrt[3]{\frac{3}{5}}\stackrel{?}{=}0$

$\sqrt[3]{\frac{9}{5}-\frac{36}{5}+6}-\sqrt[3]{\frac{3}{5}}\stackrel{?}{=}0$

$\sqrt[3]{\frac{3}{5}}-\sqrt[3]{\frac{3}{5}}\stackrel{?}{=}0$

$0=0$ ✓

$\sqrt[3]{5(2)^2-12(2)+6}-\sqrt[3]{2}\stackrel{?}{=}0$

$\sqrt[3]{20-24+6}-\sqrt[3]{2}\stackrel{?}{=}0$

$\sqrt[3]{2}-\sqrt[3]{2}\stackrel{?}{=}0$

$0=0$ ✓

Solution set: $\left\{\frac{3}{5},2\right\}$

1.6 Example 7(a) Solving Equations Quadratic in Form (page 140)

Solve $(x-3)^{1/2}-6(x-3)^{1/4}+8=0$.

Let $u=(x-3)^{1/4}$. Then $u^2=\left[(x-3)^{1/4}\right]^2=(x-3)^{1/2}$.

$u^2-6u+8=0$ — Substitute.

$(u-4)(u-2)=0$ — Factor.

$u-4=0$ or $u-2=0$ — Zero-factor property

$u=4$ or $u=2$ — Proposed solutions

1.6 Example 7(a) Solving Equations Quadratic in Form (page 140)

Solve for x by replacing u with $(x-3)^{1/4}$.

$(x-3)^{1/4}=4$	$(x-3)^{1/4}=2$	Substitute.
$x-3=4^4$	$x-3=2^4$	Raise both sides to the fourth power.
$x-3=256$	$x-3=16$	
$x=259$	$x=19$	Proposed solutions

1.6 Example 7(a) Solving Equations Quadratic in Form (cont.)

Now check.

$(x-3)^{1/2}-6(x-3)^{1/4}+8=0$

$(259-3)^{1/2}-6(259-3)^{1/4}+8\stackrel{?}{=}0$

$(256)^{1/2}-6(256)^{1/4}+8\stackrel{?}{=}0$

$16-6(4)+8\stackrel{?}{=}0$

$0=0$ ✓

$(19-3)^{1/2}-6(19-3)^{1/4}+8\stackrel{?}{=}0$

$(16)^{1/2}-6(16)^{1/4}+8\stackrel{?}{=}0$

$4-6(2)+8\stackrel{?}{=}0$

$0=0$ ✓

Solution set: {19, 259}

1.6 Example 7(b) Solving Equations Quadratic in Form (page 140)

Solve $15x^{-2}-4x^{-1}=3$.

$15x^{-2}-4x^{-1}-3=0$ — Subtract 3.

Let $u=x^{-1}$. Then $u^2=x^{-2}$.

$15u^2-4u-3=0$ — Substitute.

$(5u-3)(3u+1)=0$ — Factor.

$5u-3=0$ or $3u+1=0$ — Zero-factor property

$u=\frac{3}{5}$ or $u=-\frac{1}{3}$ — Proposed solutions

1.6 Example 7(b) Solving Equations Quadratic in Form (cont.)

Solve for x by replacing u with x^{-1}.

$x^{-1} = \frac{3}{5} \Rightarrow x = \frac{5}{3}$ $\qquad$ $x^{-1} = -\frac{1}{3} \Rightarrow x = -3$

Now check.

$$15\left(\frac{5}{3}\right)^{-2} - 4\left(\frac{5}{3}\right)^{-1} \stackrel{?}{=} 3 \qquad 15(-3)^{-2} - 4(-3)^{-1} \stackrel{?}{=} 3$$

$$15\left(\frac{3}{5}\right)^{2} - 4\left(\frac{3}{5}\right) \stackrel{?}{=} 3 \qquad 15\left(-\frac{1}{3}\right)^{2} - 4\left(-\frac{1}{3}\right) \stackrel{?}{=} 3$$

$$15\left(\frac{9}{25}\right) - \frac{12}{5} \stackrel{?}{=} 3 \qquad \frac{5}{3} + \frac{4}{3} \stackrel{?}{=} 3$$

$$3 = 3 \checkmark \qquad 3 = 3 \checkmark$$

Solution set: $\left\{\frac{5}{3}, -3\right\}$

1.6 Example 8 Solving Equations Quadratic in Form (page 141)

Solve $18x^4 - 29x^2 + 3 = 0$.

Let $u = x^2$. Then $u^2 = x^4$.

$18u^2 - 29u + 3 = 0$	Substitute.
$(2u - 3)(9u - 1) = 0$	Factor.
$2u - 3 = 0$ or $9u - 1 = 0$	Zero-factor property
$u = \frac{3}{2}$ or $u = \frac{1}{9}$	Proposed solutions

1.6 Example 8 Solving Equations Quadratic in Form (cont.)

Solve for x by replacing u with x^2.

$x^2 = \frac{3}{2}$ $\qquad$ or $x^2 = \frac{1}{9}$

$x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$ or $x = \pm\sqrt{\frac{1}{9}} = \pm\frac{1}{3}$

Now check. $\quad 18x^4 - 29x^2 + 3 = 0$

$$18\left(\frac{\sqrt{6}}{2}\right)^4 - 29\left(\frac{\sqrt{6}}{2}\right)^2 + 3 \stackrel{?}{=} 0 \qquad 18\left(\frac{1}{3}\right)^4 - 29\left(\frac{1}{3}\right)^2 + 3 \stackrel{?}{=} 0$$

$$18\left(\frac{36}{16}\right) - 29\left(\frac{6}{4}\right) + 3 \stackrel{?}{=} 0 \qquad 18\left(\frac{1}{81}\right) - 29\left(\frac{1}{9}\right) + 3 \stackrel{?}{=} 0$$

$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

Similarly, check $x = -\frac{\sqrt{6}}{2}$ and $x = -\frac{1}{3}$.

Solution set: $\left\{\pm\frac{\sqrt{6}}{2}, \pm\frac{1}{3}\right\}$

1.6 Example 9 Solving an Equation that Leads to One Quadratic in Form (page 142)

Solve $x = (10x^2 - 24)^{1/4}$.

$x^4 = 10x^2 - 24$	Raise each side to the fourth power.
$x^4 - 10x^2 + 24 = 0$	Standard form

Let $u = x^2$, then substitute:

$u^2 - 10u + 24 = 0$

$(u - 6)(u - 4) = 0 \Rightarrow u = 6$ or $u = 4$

Now solve for x.

$x^2 = 6 \Rightarrow x = \pm\sqrt{6}$ $\qquad$ $x^2 = 4 \Rightarrow x = \pm 2$

1.6 Example 9 Solving an Equation that Leads to One Quadratic in Form (cont.)

Now check. $\quad x = (10x^2 - 24)^{1/4}$.

$$-\sqrt{6} \stackrel{?}{=} \left(10(-\sqrt{6})^2 - 24\right)^{1/4} \qquad \sqrt{6} \stackrel{?}{=} \left(10(\sqrt{6})^2 - 24\right)^{1/4}$$

$$-\sqrt{6} \stackrel{?}{=} (36)^{1/4} \qquad \sqrt{6} \stackrel{?}{=} (36)^{1/4}$$

$-\sqrt{6} \neq \sqrt{6}$ Not a solution $\qquad$ $\sqrt{6} = \sqrt{6} \checkmark$

$$-2 \stackrel{?}{=} \left(10(-2)^2 - 24\right)^{1/4} \qquad 2 \stackrel{?}{=} \left(10(2)^2 - 24\right)^{1/4}$$

$$-2 \stackrel{?}{=} (16)^{1/4} \qquad 2 \stackrel{?}{=} (16)^{1/4}$$

$-2 \neq 2$ Not a solution $\qquad$ $2 = 2 \checkmark$

Solution set: $\{\sqrt{6}, 2\}$

1.7 Inequalities

Linear Inequalities Three-Part Inequalities
Rational Inequalities

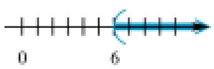
1.7 Example 1 Solving a Linear Inequality (page 146)

$$-2x + 7 < -5$$
$$-2x + 7 - 7 < -5 - 7 \quad \text{Subtract 7.}$$
$$-2x < -12$$
$$\frac{-2x}{-2} > \frac{-12}{-2}$$

Divide by –2. Reverse the direction of the inequality symbol when multiplying or dividing by a negative number.

$$x > 6$$

Solution set: $\{x|x > 6\}$

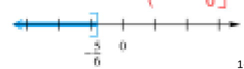


1.7 Example 2 Solving a Linear Inequality (page 147)

$$3 - 4x \geq 2x + 8$$
$$3 - 4x - 8 \geq 2x + 8 - 8 \quad \text{Subtract 8.}$$
$$-5 - 4x \geq 6x$$
$$-5 - 4x + 4x \geq 6x + 4x \quad \text{Add } 4x.$$
$$-5 \geq 6x$$
$$\frac{-5}{6} \geq \frac{6x}{6} \quad \text{Divide by 6.}$$
$$-\frac{5}{6} \geq x \Rightarrow x \leq -\frac{5}{6}$$

Write the solution set in interval notation and graph it.

Solution set: $\left(-\infty, -\frac{5}{6}\right]$



1.7 Example 3 Solving a Three-Part Inequality (page 148)

$$1 \leq 6x - 8 \leq 4$$
$$1 + 8 \leq 6x - 8 + 8 \leq 4 + 8 \quad \text{Add 8.}$$
$$9 \leq 6x \leq 12$$
$$\frac{9}{6} \leq \frac{6x}{6} \leq \frac{12}{6} \quad \text{Divide by 6.}$$
$$\frac{3}{2} \leq x \leq 2$$

Write the solution set in interval notation and graph it.

Solution set: $\left[\frac{3}{2}, 2\right]$

1.7 Example 4 Solving the Break-Even Point (page 148)

If the revenue and cost of a certain product are given by $R = 45x$ and $C = 30x + 5250$, where x is the number of units produced and sold, at what production level does R at least equal C?

Set $R \geq C$ and solve for x.

$$45x \geq 30x + 5250$$
$$15x \geq 5250 \quad \text{Subtract } 30x.$$
$$x \geq 350 \quad \text{Divide by 15.}$$

The break-even point is at $x = 350$.

This product will at least break even only if the number of units produced and sold is in the interval $[350, \infty)$.

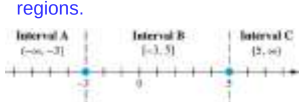
1.7 Example 5 Solving a Quadratic Inequality (page 149)

Solve $x^2 - 2x - 15 \leq 0$.

Step 1: Find the values of x that satisfy $x^2 - 2x - 15 = 0$.

$$x^2 - 2x - 15 = 0$$
$$(x - 5)(x + 3) = 0$$
$$x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 5 \quad \text{or} \quad x = -3$$

Step 2: The two numbers divide a number line into three regions.



Use closed dots since the inequality symbol includes *equality.*

1.7 Example 5 Solving a Quadratic Inequality (cont.)

Step 3: Choose a test value to see if it satisfies the inequality.

Interval	Test Value	Is $x^2 - 2x - 15 \leq 0$ True or False?
A: $(-\infty, -3]$	–5	$(-5)^2 - 2(-5) - 15 \stackrel{?}{\leq} 0$ $20 \leq 0$ False
B: $[-3, 5]$	0	$0^2 - 2(0) - 15 \stackrel{?}{\leq} 0$ $-15 \leq 0$ True
C: $[5, \infty)$	6	$6^2 - 2(6) - 15 \stackrel{?}{\leq} 0$ $9 \leq 0$ False

The values in interval B make the inequality true.

Solution set: [–3, 5]

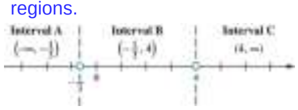
1.7 **Example 6** Solving a Quadratic Inequality (page 150)

Solve $3x^2 - 11x - 4 > 0$.

Step 1: Find the values of *x* that satisfy $3x^2 - 11x - 4 = 0$.

$$3x^2 - 11x - 4 = 0$$
$$(3x+1)(x-4) = 0$$
$$3x + 1 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -\tfrac{1}{3} \quad \text{or} \quad x = 4$$

Step 2: The two numbers divide a number line into three regions.



Use open dots since the inequality symbol does not include *equality.*

1.7 **Example 6** Solving a Quadratic Inequality (cont.)

Step 3: Choose a test value to see if it satisfies the inequality.

Interval	Test Value	Is $3x^2 - 11x - 4 > 0$ True or False?
A: $\left(-\infty, -\frac{1}{3}\right)$	−1	$3(-1)^2 - 11(-1) - 4 \overset{?}{>} 0$ $10 > 0$ True
B: $\left(-\frac{1}{3}, 4\right)$	0	$3(0)^2 - 11(0) - 4 \overset{?}{>} 0$ $-4 > 0$ False
C: $(4, \infty)$	5	$3(5)^2 - 11(5) - 4 \overset{?}{>} 0$ $16 > 0$ True

The values in intervals A and C make the inequality true.

Solution set: $\left(-\infty, -\frac{1}{3}\right) \cup (4, \infty)$

1.7 **Example 7** Solving a Problem Involving the Height of a Projectile (page 151)

If an object is launched from ground level with an initial velocity of 144 ft per sec, its height in feet *t* seconds after launching is *s* feet, where

$$s = -16t^2 + 144t.$$

When will the object be greater than 128 ft above ground level?

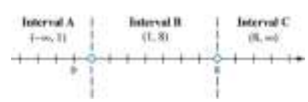
$$-16t^2 + 144t > 128 \qquad \text{Set } s \text{ greater than 128.}$$
$$-16t^2 + 144t - 128 > 0 \qquad \text{Subtract 128.}$$
$$t^2 - 9t + 8 < 0 \qquad \text{Divide by –16.}$$

1.7 **Example 7** Solving a Problem Involving the Height of a Projectile (cont.)

Step 1: Solve the corresponding equation.

$$t^2 - 9t + 8 = 0$$
$$(t-8)(t-1) = 0 \qquad \text{Factor.}$$
$$t = 8 \quad \text{or} \quad t = 1 \qquad \text{Zero-factor property}$$

Step 2: The two numbers divide a number line into three regions.



Use open dots since the inequality symbol does not include *equality.*

1.7 **Example 7** Solving a Problem Involving the Height of a Projectile (cont.)

Step 3: Choose a test value to see if it satisfies the inequality.

Interval	Test Value	Is $-16t^2 + 144t > 128$ True or False?
A: $(-\infty, 1)$	0	$-16(0)^2 + 144(0) \overset{?}{>} 128$ $0 > 128$ False
B: $(1, 8)$	2	$-16(2)^2 + 144(2) \overset{?}{>} 128$ $224 > 128$ True
C: $(8, \infty)$	10	$-16(10)^2 + 144(10) \overset{?}{>} 128$ $-160 > 128$ False

The values in interval B make the inequality true, so the solution set is (1, 8).

The object will be greater than 128 ft above ground level between 1 and 8 seconds after it is launched.

1.7 **Example 8** Solving a Rational Inequality (page 151)

Solve $\dfrac{6}{x-3} \ge 4.$

Step 1:

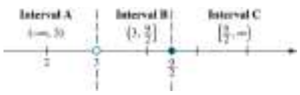
$$\frac{6}{x-3} - 4 \ge 0 \qquad \text{Subtract 4.}$$
$$\frac{6}{x-3} - \frac{4(x-3)}{x-3} \ge 0 \qquad x - 3 \text{ is the common denominator.}$$
$$\frac{6 - 4x + 12}{x-3} \ge 0 \qquad \text{Write as a single fraction.}$$
$$\frac{18 - 4x}{x-3} \ge 0 \qquad \text{Combine terms in the numerator.}$$

1.7 Example 8 Solving a Rational Inequality (cont.)

Step 2: The quotient changes sign only where *x*-values make the numerator or denominator 0.

$$18 - 4x = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = \frac{9}{2} \quad \text{or} \quad x = 3$$

The values $\frac{9}{2}$ and 3 divide the number line into three regions. Use an open circle on 3 because it makes the denominator equal 0.



1.7 Example 8 Solving a Rational Inequality (cont.)

Step 3: Choose a test value to see if it satisfies the inequality.

Interval	Test Value	Is $\frac{8}{x-3} \geq 4$ True or False?
A: $(-\infty, 3)$	0	$\frac{8}{0-3} \overset{?}{\geq} 4$ $-2 > 4$ False
B: $\left(3, \frac{9}{2}\right]$	4	$\frac{8}{4-3} \overset{?}{\geq} 4$ $8 > 4$ True
C: $\left[\frac{9}{2}, \infty\right)$	5	$\frac{8}{5-3} \overset{?}{\geq} 4$ $3 > 4$ False

The values in interval B make the inequality true.

Solution set: $\left(3, \frac{9}{2}\right]$

1.7 Example 9 Solving a Rational Inequality (page 153)

Solve $\frac{3x+1}{2x-3} < 4.$

Step 1:

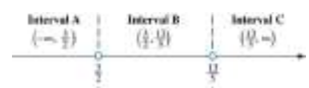
$\frac{3x+1}{2x-3} - 4 < 0$	Subtract 4.
$\frac{3x+1}{2x-3} - \frac{4(2x-3)}{2x-3} < 0$	$2x - 3$ is the common denominator.
$\frac{3x+1-8x+12}{2x-3} < 0$	Write as a single fraction.
$\frac{-5x+13}{2x-3} < 0$	Combine terms in the numerator.

1.7 Example 9 Solving a Rational Inequality (cont.)

Step 2: The quotient changes sign only where *x*-values make the numerator or denominator 0.

$$-5x + 13 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$x = \frac{13}{5} \quad \text{or} \quad x = \frac{3}{2}$$

The values $\frac{3}{2}$ and $\frac{13}{5}$ divide the number line into three regions. Use open circles because equality is not included.



1.7 Example 9 Solving a Rational Inequality (cont.)

Step 3: Choose a test value to see if it satisfies the inequality.

Interval	Test Value	Is $\frac{3x+1}{2x-3} < 4$ True or False?
A: $\left(-\infty, \frac{3}{2}\right)$	0	$\frac{3(0)+1}{2(0)-3} \overset{?}{<} 4$ $-\frac{1}{3} < 4$ True
B: $\left(\frac{3}{2}, \frac{13}{5}\right)$	2	$\frac{3(2)+1}{2(2)-3} \overset{?}{<} 4$ $7 < 4$ False
C: $\left(\frac{13}{5}, \infty\right)$	5	$\frac{3(5)+1}{2(5)-3} \overset{?}{<} 4$ $\frac{16}{7} < 4$ True

The values in intervals A and C make the inequality true.

Solution set: $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{13}{5}, \infty\right)$

1.8 Absolute Value Equations and Inequalities

Absolute Value Equations   Absolute Value Inequalities   Special Cases   Absolute Value Models for Distance and Tolerance

1.8 Example 1(a) Solving Absolute Value Equations (page 159)

$$|9-4x|=7$$

$9-4x=7$ or $9-4x=-7$ — Property 1

$-4x=-2$ or $-4x=-16$ — Subtract 9.

$x=\frac{1}{2}$ or $x=4$ — Divide by –4.

Now check.

$\left\|9-4\left(\frac{1}{2}\right)\right\|\overset{?}{=}7$	$\|9-4(4)\|\overset{?}{=}7$
$\|9-2\|\overset{?}{=}7$	$\|9-16\|\overset{?}{=}7$
$\|7\|\overset{?}{=}7$	$\|-7\|\overset{?}{=}7$
$7=7$ ✓	$7=7$ ✓

Solution set: $\left\{\frac{1}{2},4\right\}$

1.8 Example 1(b) Solving Absolute Value Equations (page 159)

$$|3x+2|=|x-5|$$

$3x+2=x-5$ or $3x+2=-(x-5)$ — Property 2

$2x=-7$ or $3x+2=-x+5$

$x=-\frac{7}{2}$ or $4x=3 \Rightarrow x=\frac{3}{4}$

Now check.

$\left\|3\left(-\frac{7}{2}\right)+2\right\|\overset{?}{=}\left\|-\frac{7}{2}-5\right\|$	$\left\|3\left(\frac{3}{4}\right)+2\right\|\overset{?}{=}\left\|\frac{3}{4}-5\right\|$
$\left\|-\frac{17}{2}\right\|\overset{?}{=}\left\|-\frac{17}{2}\right\|$	$\left\|\frac{17}{4}\right\|\overset{?}{=}\left\|-\frac{17}{4}\right\|$
$\frac{17}{2}=\frac{17}{2}$ ✓	$\frac{17}{4}=\frac{17}{4}$ ✓

Solution set: $\left\{-\frac{7}{2},\frac{3}{4}\right\}$

1.8 Example 2(a) Solving Absolute Value Inequalities (page 160)

$$|4x-6|<10$$

$-10<4x-6<10$ — Property 3

$-4<4x<16$ — Add 6.

$-1<x<4$ — Divide by 4.

Solution set: (–1, 4)

1.8 Example 2(b) Solving Absolute Value Inequalities (page 160)

$$|4x-6|>10$$

$4x-6<-10$ or $4x-6>10$ — Property 4

$4x<-4$ or $4x>16$ — Add 6.

$x<-1$ or $x>4$ — Divide by 4.

Solution set: $(-\infty,-1)\cup(4,\infty)$

1.8 Example 3 Solving Absolute Value Inequalities Requiring a Transformation (page 161)

$$|5-8x|+6\ge 14$$

$|5-8x|\ge 8$ — Subtract 6.

$5-8x\le -8$ or $5-8x\ge 8$ — Property 4

$-8x\le -13$ or $-8x\ge 3$ — Subtract 5.

$x\ge\frac{13}{8}$ or $x\le-\frac{3}{8}$ — Divide by –8. Reverse the direction of the inequality symbol.

Solution set: $\left(-\infty,-\frac{3}{8}\right]\cup\left[\frac{13}{8},\infty\right)$

1.8 Example 4(a) Solving Special Cases of Absolute Value Equations and Inequalities (page 161)

$$|7x+28|=0$$

The absolute value of a number will be 0 if that number is 0.

Therefore, $|7x+28|=0$ is equivalent to $7x + 28 = 0$.

Solution set: {–4}

1.8 Example 4(b) Solving Special Cases of Absolute Value Equations and Inequalities (page 161)

$$|6x - 9| > -2$$

The absolute value of a number is always nonnegative.

Therefore, $|6x - 9| > -2$ is always true.

Solution set: $(-\infty, \infty)$

1.8 Example 4(c) Solving Special Cases of Absolute Value Equations and Inequalities (page 161)

$$|2 - 5x| \le -5$$

There is no number whose absolute value is less than –5.

Therefore, $|2 - 5x| \le -5$ is always false.

Solution set: Ø

1.8 Example 5 Using Absolute Value Inequalities to Describe Distances (page 162)

Write each statement using an absolute value inequality.

(a) *m* is no more than 9 units from 3.

This means that *m* is 9 units or less from 3. Thus the distance between *m* and 3 is less than or equal to 9, or $|m - 3| \le 9$.

(b) *t* is within .02 unit of 5.8 .

This means that *t* is less than .02 unit from 5.8. Thus the distance between *t* and 5.8 is less than .02, or $|t - 5.8| < .02$.

1.8 Example 6 Using Absolute Value to Model Tolerance (page 162)

Suppose *y* = 5*x* – 2 and we want *y* to be within .001 unit of 6. For what values of *x* will this be true?

$|y - 6| < .001$ Write an absolute value inequality

$|(5x - 2) - 6| < .001$ Substitute 5*x* – 2 for *y*.

$|5x - 8| < .001$

$-.001 < 5x - 8 < .001$ Property 3

$7.999 < 5x < 8.001$ Add 8.

$1.5998 < x < 1.6002$ Divide by 5.

Values of *x* in the interval (1.5998, 1.6002) will satisfy the condition.