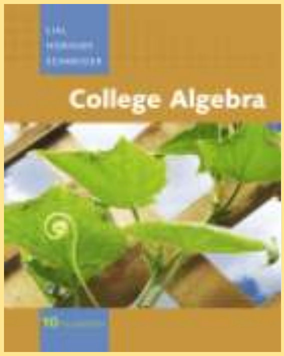
2

Graphs and Functions

Sections 2.1–2.4



2 Graphs and Functions

2.1 Rectangular Coordinates and Graphs

Ordered Pairs The Rectangular Coordinate System
The Distance Formula The Midpoint Formula
Graphing Equations

2.1 Example 1 Writing Ordered Pairs (page 182)

Type of Entertainment	Amount Spent
VHS rentals/sales	$23
DVD rentals/sales	$71
CDs	$20
theme parks	$43
sports tickets	$50
movie tickets	$30

Source: PricewaterhouseCoopers; Pollstar.

Write an ordered pair to express the relationship between

(a) VHS rentals/sales and amount spent

(VHS rentals/sales, $23)

(b) Sport tickets and amount spent

(sports tickets, $50)

2.1 Example 2 Using the Distance Formula (page 184)

Find the distance between $P(3, -5)$ and $Q(-2, 8)$.

$$d(P,Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

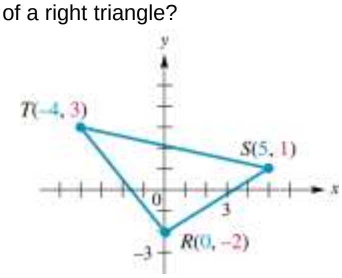
$$x_1 = 3, y_1 = -5, x_2 = -2, y_2 = 8$$

$$d(P,Q) = \sqrt{(-2-3)^2 + [8-(-5)]^2}$$

$$= \sqrt{25+169} = \sqrt{194}$$

2.1 Example 3 Determining Whether Three Points are the Vertices of a Right Triangle (page 184)

Are the points $R(0, -2)$, $S(5, 1)$ and $T(-4, 3)$ the vertices of a right triangle?



2.1 Example 3 Determining Whether Three Points are the Vertices of a Right Triangle (cont.)

Are the points $R(0, –2)$, $S(5, 1)$ and $T(– 4, 3)$ the vertices of a right triangle?

Use the distance formula to find the length of each side:

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d(R,S) = \sqrt{(5-0)^2 + [1-(-2)]^2} = \sqrt{25+9} = \sqrt{34}$$

2.1 Example 3 Determining Whether Three Points are the Vertices of a Right Triangle (cont.)

Are the points $R(0, –2)$, $S(5, 1)$ and $T(– 4, 3)$ the vertices of a right triangle?

$$d(R,T) = \sqrt{(-4-0)^2 + [3-(-2)]^2} = \sqrt{16+25} = \sqrt{41}$$

$$d(S,T) = \sqrt{(-4-5)^2 + (3-1)^2} = \sqrt{81+4} = \sqrt{85}$$

2.1 Example 3 Determining Whether Three Points are the Vertices of a Right Triangle (cont.)

Are the points $R(0, –2)$, $S(5, 1)$ and $T(– 4, 3)$ the vertices of a right triangle?

$$d(R,S) = \sqrt{34},\ d(R,T) = \sqrt{41},\ d(S,T) = \sqrt{85}$$

Use the Pythagorean Theorem to determine if the triangle is a right triangle.

The longest side has length $\sqrt{85}$.

$$\left(\sqrt{34}\right)^2 + \left(\sqrt{41}\right)^2 \overset{?}{=} \left(\sqrt{85}\right)^2$$

$$34 + 41 \neq 85$$

The triangle formed by the three points is not a right triangle.

2.1 Example 4 Determining Whether Three Points are Collinear (page 185)

Are the points $A(–2, 5)$, $B(0, 3)$, and $C(8, –5)$ collinear?

The distance between $A(–2, 5)$ and $B(0, 3)$ is

$$\sqrt{(-2-0)^2 + (5-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

The distance between $B(0, 3)$ and $C(8, –5)$ is

$$\sqrt{(8-0)^2 + (-5-3)^2} = \sqrt{64+64} = \sqrt{128} = 8\sqrt{2}$$

2.1 Example 4 Determining Whether Three Points are Collinear (cont.)

Are the points $A(–2, 5)$, $B(0, 3)$, and $C(8, –5)$ collinear?

The distance between $A(–2, 5)$ and $C(8, –5)$ is

$$\sqrt{(-2-8)^2 + [5-(-5)]^2} = \sqrt{100+100} = \sqrt{200} = 10\sqrt{2}$$

$$d(AB) = 2\sqrt{2},\ d(BC) = 8\sqrt{2},\ d(AC) = 10\sqrt{2}$$

$$d(AB) + d(BC) \overset{?}{=} d(AC)$$

$$2\sqrt{2} + 8\sqrt{2} = 10\sqrt{2}$$

Yes, the points are collinear.

2.1 Example 5(a) Using the Midpoint Formula (page 186)

Find the coordinates of the midpoint M of the segment with endpoints $(–7, –5)$ and $(–2, 13)$.

Midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$\left(\frac{-7+(-2)}{2}, \frac{-5+13}{2}\right) = \left(-\frac{9}{2}, 4\right)$$

2.1 **Example 5(b)** Using the Midpoint Formula (page 186)

Find the coordinates of the other endpoint *B* of a segment with one endpoint *A*(8, –20) and midpoint *M*(4, –4).

Let (*x*, *y*) be the coordinates of *B*. Using the midpoint formula, we have

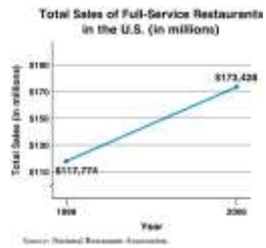
$$\left(\frac{8+x}{2}, \frac{-20+y}{2}\right) = (4, -4)$$

$$\frac{8+x}{2} = 4 \Rightarrow 8+x = 8 \Rightarrow x = 0$$

$$\frac{-20+y}{2} = -4 \Rightarrow -20+y = -8 \Rightarrow y = 12$$

The coordinates of *B* are (0, 12).

2.1 **Example 6** Applying the Midpoint Formula to Data (page 186)



Use the midpoint formula to estimate the total sales in 2002. Compare this to the actual figure of $141,874 million.

2.1 **Example 6** Applying the Midpoint Formula to Data (cont.)

The endpoints of the segment are (1998, 117,774) and (2006, 173,428).

$$M = \left(\frac{1998+2006}{2}, \frac{117{,}774+173{,}428}{2}\right)$$
$$= (2002, 145{,}601)$$

Estimated total sales in 2006 are $145,601 million.

Actual amount – estimated amount
= $141,874 million – $145,601 million = –$3727 million.

The estimate of $145,601 million is $3727 million more than the actual amount.

2.1 **Example 7(a)** Finding Ordered Pairs That are Solutions to Equations (page 187)

Find three ordered pairs that are solutions to *y* = –2*x* + 5.

x	$y = -2x + 5$
–1	$y = -2(-1) + 5 = 7$
1	$y = -2(1) + 5 = 3$
3	$y = -2(3) + 5 = -1$

Three ordered pairs that are solutions are (−1, 7), (1, 3), and (3, −1).

2.1 **Example 7(b)** Finding Ordered Pairs That are Solutions to Equations (page 187)

Find three ordered pairs that are solutions to $x = \sqrt[3]{y+1}$.

y	$x = \sqrt[3]{y+1}$
–9	$x = \sqrt[3]{-9+1} = \sqrt[3]{-8} = -2$
–2	$x = \sqrt[3]{-2+1} = \sqrt[3]{-1} = -1$
7	$x = \sqrt[3]{7+1} = \sqrt[3]{8} = 2$

Three ordered pairs that are solutions are (−2, −9), (−1, −2), and (2, 7).

2.1 **Example 7(c)** Finding Ordered Pairs That are Solutions to Equations (page 187)

Find three ordered pairs that are solutions to $y = -x^2 + 1$.

x	$y = -x^2 + 1$
–2	$y = -(-2)^2 + 1 = -3$
–1	$y = -(-1)^2 + 1 = 0$
2	$y = -2^2 + 1 = -3$

Three ordered pairs that are solutions are (−2, −3), (−1, 0), and (2, −3).

2.1 Example 8(a) Graphing Equations (page 188)

Graph the equation $y = –2x + 5$.

Step 1: Find the x-intercept and the y-intercept.

$$y = -2x + 5 \qquad\qquad y = -2x + 5$$
$$0 = -2x + 5 \qquad\qquad y = -2(0) + 5$$
$$-5 = -2x \qquad\qquad y = 0 + 5 = 5 \quad y\text{-intercept}$$
$$\frac{5}{2} = x \quad x\text{-intercept}$$

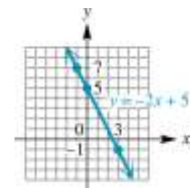
Intercepts: $\left(\frac{5}{2}, 0\right)$ and $(0, 5)$

2.1 Example 8(a) Graphing Equations (cont.)

Graph the equation $y = –2x + 5$.

Step 2: Use the other ordered pairs found in Example 7b: $(−1, 7)$, $(1, 3)$, and $(3, −1)$.

Steps 3 and 4: Plot and then connect the five points.



2.1 Example 8(b) Graphing Equations (page 188)

Graph the equation $x = \sqrt[3]{y+1}$.

Step 1: Find the x-intercept and the y-intercept.

$$y = \sqrt[3]{x+1} \qquad\qquad x = \sqrt[3]{0+1}$$
$$0 = \sqrt[3]{x+1} \qquad\qquad = \sqrt[3]{1} = 1 \quad y\text{-intercept}$$
$$0^3 = x + 1$$
$$-1 = x \quad x\text{-intercept}$$

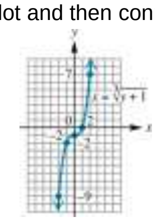
Intercepts: $(–1, 0)$ and $(0, 1)$

2.1 Example 8(b) Graphing Equations (cont.)

Graph the equation $x = \sqrt[3]{y+1}$.

Step 2: Use the other ordered pairs found in Example 7b: $(−2, −9)$, $(−1, −2)$, and $(2, 7)$.

Steps 3 and 4: Plot and then connect the five points.



2.1 Example 8(c) Graphing Equations (page 188)

Graph the equation $y = -x^2 + 1$.

Step 1: Find the x-intercept and the y-intercept.

$$0 = -x^2 + 1 \qquad\qquad y = -(0)^2 + 1$$
$$-1 = -x^2 \Rightarrow 1 = x^2 \qquad\qquad y = 1 \quad y\text{-intercept}$$
$$\pm 1 = x \quad x\text{-intercepts}$$

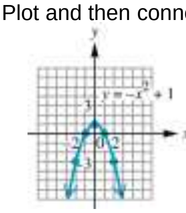
Intercepts: $(–1, 0)$, $(1, 0)$ and $(0, 1)$

2.1 Example 8(c) Graphing Equations (cont.)

Graph the equation $y = -x^2 + 1$.

Step 2: Use the other ordered pairs found in Example 7c: $(−2, −3)$ and $(2, −3)$.

Steps 3 and 4: Plot and then connect the five points.



2.2 Circles

Center-Radius Form   General Form   An Application

2.2 Example 1 Find the Center-Radius Form (page 194)

Find the center-radius form of the equation of each circle described.

$$(x-h)^2+(y-k)^2=r^2$$

(a) Center at (1, –2), radius 3

$(h, k) = (1, -2)$, $r = 3$ $(x-1)^2+[y-(-2)]^2=3^2$

$$(x-1)^2+(y+2)^2=9$$

(b) Center at (0, 0), radius 2

$(h, k) = (0, 0)$, $r = 2$ $(x-0)^2+(y-0)^2=2^2$

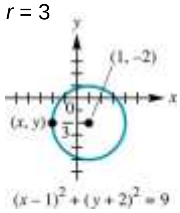
$$x^2+y^2=4$$

2.2 Example 2(a) Graphing Circles (page 194)

Graph $(x-1)^2+(y+2)^2=9$

Write the equation as $(x-1)^2+[y-(-2)]^2=3^2$

$(h, k) = (1, -2)$, $r = 3$

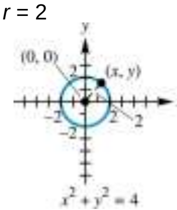


2.2 Example 2(b) Graphing Circles (page 194)

Graph $x^2+y^2=4$

Write the equation as $x^2+y^2=2^2$

$(h, k) = (0, 0)$, $r = 2$



2.2 Example 3 Finding the Center and Radius by Completing the Square (page 196)

Show that $x^2+4x+y^2-8y-44=0$ has a circle as its graph. Find the center and radius.

Complete the square twice, once for *x* and once for *y*.

$$x^2+4x+y^2-8y-44=0$$

$$(x^2+4x+\quad)+(y^2-8y+\quad)=44$$

$$\left[\frac{1}{2}(4)\right]^2=4 \qquad \left[\frac{1}{2}(-8)\right]^2=16$$

$$(x^2+4x+4)+(y^2-8y+16)=44+4+16$$

$$(x+2)^2+(y-4)^2=64$$

2.2 Example 3 Finding the Center and Radius by Completing the Square (cont.)

Show that $x^2+4x+y^2-8y-44=0$ has a circle as its graph. Find the center and radius.

$$(x+2)^2+(y-4)^2=64$$

Since 64 > 0, the graph is a circle.

Center: (–2, 4)   Radius: 8

2.2 **Example 4** Finding the Center and Radius by Completing the Square (page 196)

Show that $2x^2+2y^2+2x-6y=45$ has a circle as its graph. Find the center and radius.

Group the terms and factor:

$$\left(2x^2+2x\right)+\left(2y^2-6y\right)=45$$

$$2\left(x^2+x\right)+2\left(y^2-3y\right)=45$$

2.2 **Example 4** Finding the Center and Radius by Completing the Square (cont.)

Complete the squares on *x* and *y,* then factor:

$$2\left(x^2+x\right)+2\left(y^2-3y\right)=45$$

$$\left[\frac{1}{2}(1)\right]^2=\frac{1}{4} \qquad \left[\frac{1}{2}(-3)\right]^2=\frac{9}{4}$$

$$2\left(x^2+x+\frac{1}{4}\right)+2\left(y^2-3y+\frac{9}{4}\right)=45+2\left(\frac{1}{4}\right)+2\left(\frac{9}{4}\right)$$

$$2\left(x+\frac{1}{2}\right)^2+2\left(y-\frac{3}{2}\right)^2=50$$

2.2 **Example 4** Finding the Center and Radius by Completing the Square (cont.)

Divide both sides by 2:

$$2\left(x+\frac{1}{2}\right)^2+2\left(y-\frac{3}{2}\right)^2=50$$

$$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=25$$

Since 25 > 0, the graph of $x^2+4x+y^2-8y-44=0$ is a circle.

Center: $\left(-\frac{1}{2},\frac{3}{2}\right)$ Radius: 5

2.2 **Example 5** Determining Whether a Graph is a Point or Nonexistent (page 197)

The graph of the equation $x^2-6x+y^2+2y+12=0$ is either a point or is nonexistent. Which is it?

Complete the square twice, once for *x* and once for *y*.

$$\left(x^2-6x\right)+\left(y^2+2y\right)=-12$$

$$\left[\frac{1}{2}(-6)\right]^2=9 \qquad \left[\frac{1}{2}(2)\right]^2=1$$

$$\left(x^2-6x+9\right)+\left(y^2+2y+1\right)=-12+9+1$$

2.2 **Example 5** Determining Whether a Graph is a Point or Nonexistent (cont.)

Factor.

$$\left(x^2-6x+9\right)+\left(y^2+2y+1\right)=-12+9+1$$

$$(x-3)^2+(y+1)^2=-2$$

Since –2 < 0, the graph of $x^2-6x+y^2+2y+12=0$ is nonexistent.

2.2 **Example 6** Locating the Epicenter of an Earthquake (page 197)

If three receiving stations at (1, 4), (–6, 0) and (5, –2) record distances to an earthquake epicenter of 4 units, 5 units, and 10 units, respectively, show algebraically that the epicenter lies at (–3, 4).

Determine the equation for each circle and then substitute *x* = –3 and *y* = 4 to see if the point lies on all three graphs.

2.2 **Example 6** Locating the Epicenter of an Earthquake (cont.)

Station A: center (1, 4), radius 4

$$(x-1)^2+(y-4)^2=16$$
$$(-3-1)^2+(4-4)^2=16$$
$$4^2+0^2=16 \Rightarrow 16=16$$

(–3, 4) lies on the circle.

2.2 **Example 6** Locating the Epicenter of an Earthquake (cont.)

Station B: center (–6, 0), radius 5

$$(x+6)^2+y^2=25$$
$$(-3+6)^2+(4)^2=25$$
$$9+16=25 \Rightarrow 25=25$$

(–3, 4) lies on the circle.

2.2 **Example 6** Locating the Epicenter of an Earthquake (cont.)

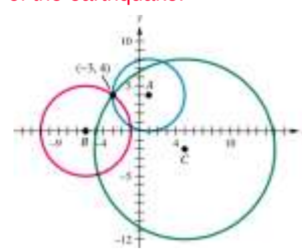
Station C: center (5, –2), radius 10

$$(x-5)^2+(y+2)^2=100$$
$$(-3-5)^2+(4+2)^2=100$$
$$64+36=100 \Rightarrow 100=100$$

(–3, 4) lies on the circle.

2.2 **Example 6** Locating the Epicenter of an Earthquake (cont.)

Since (–3, 4) lies on all three circles, it is the epicenter of the earthquake.



2.3 Functions

Relations and Functions Domain and Range Determining Functions from Graphs or Equations Function Notation Increasing, Decreasing, and Constant Functions

2.3 **Example 1** Deciding Whether Relations Define Functions (page 202)

Decide whether the relation determines a function.

$$M=\{(-4,0),(-3,1),(3,1)\}$$

M is a function because each distinct *x*-value has exactly one *y*-value.

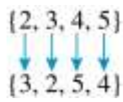
$$\{-4, -3, 3\}$$
$$\{\ 0,\ 1\}$$

2.3 **Example 1** Deciding Whether Relations Define Functions (cont.)

Decide whether the relation determines a function.

$$N = \{(2,3),(3,2),(4,5),(5,4)\}$$

N is a function because each distinct *x*-value has exactly one *y*-value.

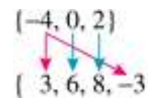


2.3 **Example 1** Deciding Whether Relations Define Functions (cont.)

Decide whether the relation determines a function.

$$P = \{(-4,3),(0,6),(2,8),(-4,-3)\}$$

P is not a function because the *x*-value –4 has two *y*-values.



2.3 **Example 2(a)** Deciding Whether Relations Define Functions (page 203)

Give the domain and range of the relation. Is the relation a function?

{(–4, –2), (–1, 0), (1, 2), (3, 5)}

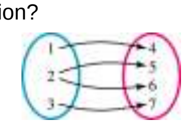
Domain: {–4, –1, 0, 3}

Range: {–2, 0, 2, 5}

The relation is a function because each *x*-value corresponds to exactly one *y*-value.

2.3 **Example 2(b)** Deciding Whether Relations Define Functions (cont.)

Give the domain and range of the relation. Is the relation a function?



Domain: {1, 2, 3}

Range: {4, 5, 6, 7}

The relation is not a function because the *x*-value 2 corresponds to two *y*-values, 5 and 6.

2.3 **Example 2(c)** Deciding Whether Relations Define Functions (cont.)

Give the domain and range of the relation. Is the relation a function?

x	*y*
–3	5
0	5
3	5
5	5

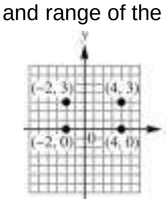
Domain: {–3, 0, 3, 5}

Range: {5}

The relation is a function because each *x*-value corresponds to exactly one *y*-value.

2.3 **Example 3(a)** Finding Domains and Ranges from Graphs (page 204)

Give the domain and range of the relation.

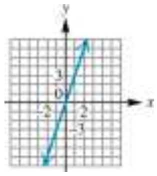


Domain: {–2, 4}

Range: {0, 3}

2.3 Example 3(b) Finding Domains and Ranges from Graphs (cont.)

Give the domain and range of the relation.

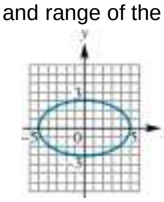


Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

2.3 Example 3(c) Finding Domains and Ranges from Graphs (cont.)

Give the domain and range of the relation.

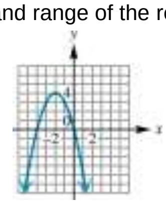


Domain: [–5, 5]

Range: [–3, 3]

2.3 Example 3(d) Finding Domains and Ranges from Graphs (cont.)

Give the domain and range of the relation.

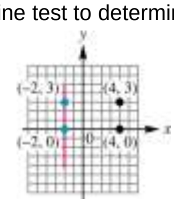


Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

2.3 Example 4(a) Using the Vertical Line Test (page 206)

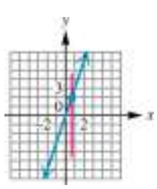
Use the vertical line test to determine if the relation is a function.



Not a function

2.3 Example 4(b) Using the Vertical Line Test (cont.)

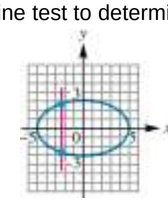
Use the vertical line test to determine if the relation is a function.



Function

2.3 Example 4(c) Using the Vertical Line Test (cont.)

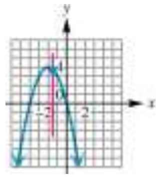
Use the vertical line test to determine if the relation is a function.



Not a function

2.3 Example 4(d) Using the Vertical Line Test (cont.)

Use the vertical line test to determine if the relation is a function.



Function

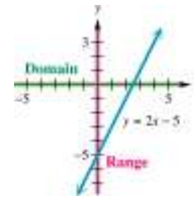
2.3 Example 5(a) Identifying Functions, Domains, and Ranges (page 206)

Determine if the relation is a function and give the domain and range.

$y = 2x - 5$

y is found by multiplying x by 2 and subtracting 5.

Each value of x corresponds to just one value of y, so the relation is a function.



Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

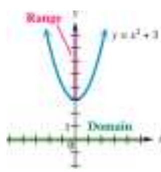
2.3 Example 5(e) Identifying Functions, Domains, and Ranges (cont.)

Determine if the relation is a function and give the domain and range.

$y = x^2 + 3$

y is found by squaring x by 2 and adding 3.

Each value of x corresponds to just one value of y, so the relation is a function.



Domain: $(-\infty, \infty)$ Range: $[3, \infty)$

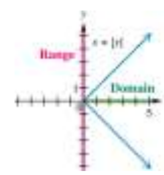
2.3 Example 5(c) Identifying Functions, Domains, and Ranges (cont.)

Determine if the relation is a function and give the domain and range.

$x = |y|$

For any choice of x in the domain, there are two possible values for y.

The relation is not a function.



Domain: $[0, \infty)$ Range: $(-\infty, \infty)$

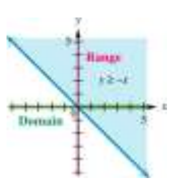
2.3 Example 5(d) Identifying Functions, Domains, and Ranges (cont.)

Determine if the relation is a function and give the domain and range.

$y \geq -x$

If $x = 1$, for example, there are many values of y that satisfy the relation.

The relation is not a function.



Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

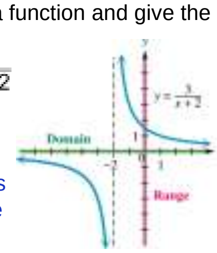
2.3 Example 5(e) Identifying Functions, Domains, and Ranges (cont.)

Determine if the relation is a function and give the domain and range.

$$y = \frac{3}{x+2}$$

y is found by dividing 3 by $x + 2$.

Each value of x corresponds to just one value of y, so the relation is a function.



Domain: $(-\infty, -2) \cup (-2, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$

2.3 **Example 6** Using Function Notation (page 209)

Let $f(x) = -x^2 - 6x + 4$ and $g(x) = 3x + 1$.
Find $f(-3)$, $f(r)$, and $g(r + 2)$.

$$f(-3) = -(-3)^2 - 6(-3) + 4 = -9 + 18 + 4 = 13$$

$$f(r) = -r^2 - 6r + 4$$

$$g(r+2) = 3(r+2) + 1 = 3r + 6 + 1 = 3r + 7$$

2.3 **Example 7** Using Function Notation (page 209)

Find $f(-1)$ for each function.

(a) $f(x) = 2x^2 - 9$

$f(-1) = 2(-1)^2 - 9 = -7$

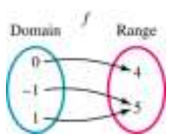
(b) $f = \{(-4, 0), (-1, 6), (0, 8), (2, -2)\}$

$f(-1) = 6$

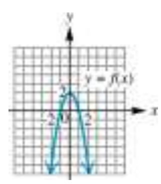
2.3 **Example 7** Using Function Notation (cont.)

Find $f(-1)$ for each function.

(c)



(d)



$f(-1) = 5$

$f(-1) = 0$

2.3 **Example 8(a)** Writing Equations Using Function Notation (page 210)

Assume that y is a function of x. Write the equation using function notation. Then find $f(-5)$ and $f(b)$.

$$y = x^2 + 2x - 3$$

$f(x) = x^2 + 2x - 3$

$f(-5) = (-5)^2 + 2(-5) - 3 = 12$

$f(b) = b^2 + 2b - 3$

2.3 **Example 8(b)** Writing Equations Using Function Notation (page 210)

Assume that y is a function of x. Write the equation using function notation. Then find $f(-5)$ and $f(b)$.

$$2x - 3y = 6$$

Solve for y:

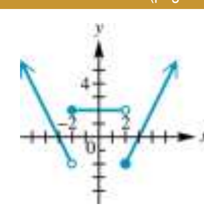
$$2x - 3y = 6 \Rightarrow -3y = -2x + 6 \Rightarrow y = \frac{2}{3}x - 2$$

$$f(x) = \frac{2}{3}x - 2$$

$$f(-5) = \frac{2}{3}(-5) - 2 = -\frac{16}{3} \qquad f(b) = \frac{2}{3}b - 2$$

2.3 **Example 9** Determining Intervals Over Which a Function is Increasing, Decreasing, or Constant (page 212)

The figure is the graph of a function. Determine the intervals over which the function is increasing, decreasing, or constant.



Increasing on $[2, \infty)$

Decreasing on $(-\infty, -2)$

Constant on $[-2, 2)$

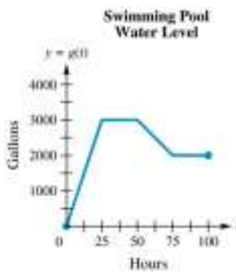
2.3 **Example 10** Interpreting a Graph (page 212)

Over what period of time is the water level changing most rapidly?

from 0 hours to 25 hours

After how many hours does the water level start to decrease?

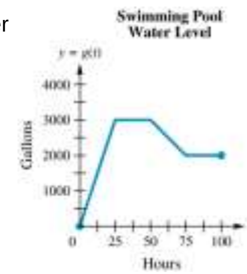
after 50 hours



2.3 **Example 10** Interpreting a Graph (cont.)

How many gallons of water are in the pool after 75 hours?

2000 gallons



2.4 Linear Functions

Graphing Linear Functions   Standard From $Ax + By = C$

Slope   Average Rate of Change   Linear Models

2.4 **Example 1** Graphing a Linear Function Using Intercepts (page 218)

Graph $f(x) = \frac{3}{2}x + 6$. Give the domain and range.

Find the *x*-intercept by letting *f*(*x*) = 0 and solving for *x*.

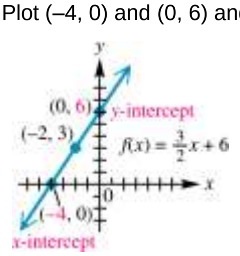
$$0 = \frac{3}{2}x + 6 \Rightarrow -6 = \frac{3}{2}x \Rightarrow x = -4$$

Find the *y*-intercept by finding *f*(0).

$$f(0) = \frac{3}{2}(0) + 6 = 6$$

2.4 **Example 1** Graphing a Linear Function Using Intercepts (cont.)

Plot (–4, 0) and (0, 6) and join with a straight line.



Verify that (–2, 3) lies on the graph.

$$3 \stackrel{?}{=} \frac{3}{2}(-2) + 6$$

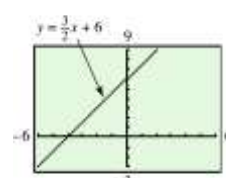
$$3 \stackrel{?}{=} -3 + 6$$

$$6 = 6 \checkmark$$

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

2.4 **Example 1** Graphing a Linear Function Using Intercepts (cont.)

Corresponding calculator graph:

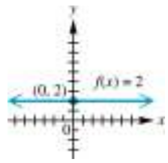


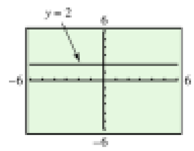
2.4 **Example 2** Graphing a Horizontal Line (page 219)

Graph $f(x) = 2$. Give the domain and range.

Since $f(x)$ always equals 2, the value of y can never be 0. So, there is no x-intercept. The line is parallel to the x-axis.

The y-intercept is 2.





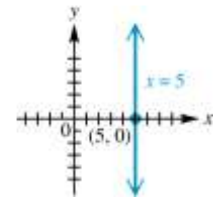
Domain: $(-\infty, \infty)$ Range: $\{2\}$

2.4 **Example 3** Graphing a Vertical Line (page 219)

Graph $x = 5$. Give the domain and range.

Since x always equals 5, the value of x can never be 0. So, there is no y-intercept. The line is parallel to the y-axis. The relation is ***not*** a function.

The x-intercept is 5.



Domain: $\{5\}$ Range: $(-\infty, \infty)$

2.4 **Example 4** Graphing $Ax + By = C$ with $C = 0$ (page 220)

Graph $3x + 4y = 0$. Give the domain and range.

Find the intercepts.

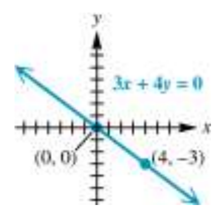
x-intercept	y-intercept
$3x + 4(0) = 0$	$3(0) + 4y = 0$
$x = 0$	$y = 0$

There is only one intercept, the origin, (0, 0).
Choose $x = 4$ to find a second point.

$$3(4) + 4y = 0$$
$$12 + 4y = 0$$
$$y = -3$$

2.4 **Example 4** Graphing $Ax + By = C$ with $C = 0$ (cont.)

Graph the points (0, 0) and (4, –3) and join with a straight line.



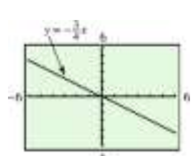
Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

2.4 **Example 4** Graphing $Ax + By = C$ with $C = 0$ (cont.)

Graphing calculator solution

Solve the equation for y to graph the equation on a graphing calculator.

$$3x + 4y = 0$$
$$4y = -3x$$
$$y = -\frac{3}{4}x$$



2.4 **Example 5** Finding Slopes with the Slope Formula (page 221)

Find the slope of the line through the given points.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

(a) (–2, 4), (2, –6) $\quad m = \frac{4-(-6)}{-2-2} = \frac{10}{-4} = -\frac{5}{2}$

(b) (–3, 8), (5, 8) $\quad m = \frac{8-8}{5-(-3)} = \frac{0}{8} = 0$

(c) (–4, –10), (–4, 10) $\quad m = \frac{-10-10}{-4-(-4)} = \frac{-20}{0} \Rightarrow$

the slope is undefined

2.4 Example 6 Finding the Slope from an Equation (page 222)

Find the slope of the line $y = 5x - 3$.

Find any two ordered pairs that are solutions of the equation.

If $x = 0$, then $y = 5(0) - 3 = -3$.

If $x = 1$, then $y = 5(1) - 3 = 2$.

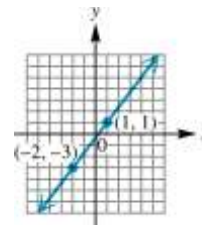
$$m = \frac{2-(-3)}{1-0} = 5$$

2.4 Example 7 Graphing a Line Using a Point and the Slope (page 222)

Graph the line passing through (–2, –3) and having slope $\frac{4}{3}$.

Plot the point (–2, –3).

The slope is $\frac{4}{3}$, so a change of 3 units horizontally produces a change of 4 units vertically. This gives the point (1, 1).



Join the points with a straight line to complete the graph.

2.4 Example 8 Interpreting Slope as Average Rate of Change (page 223)

In 1997, sales of VCRs numbered 16.7 million. In 2002, estimated sales of VCRs were 13.3 million. Find the average rate of change in VCR sales, in millions, per year. Graph as a line segment, and interpret the result.

The average rate of change per year is

$$\frac{13.3-16.7}{2002-1997} = \frac{-3.4}{5} = -0.68 \text{ million}$$

2.4 Example 8 Interpreting Slope as Average Rate of Change (cont.)

In 1997, sales of VCRs numbered 16.7 million. In 2002, estimated sales of VCRs were 13.3 million. Find the average rate of change in VCR sales, in millions, per year. Graph as a line segment, and interpret the result.

The graph confirms that the line through the ordered pairs fall from left to right, and therefore has negative slope.

Sales of VCRs decreased by an average of 0.68 million each year from 1997 to 2002.

2.4 Example 9 Writing Linear Cost, Revenue, and Profit Functions (page 224)

Assume that the cost to produce an item is a linear function and all items produced are sold. The fixed cost is $2400, the variable cost per item is $120, and the item sells for $150. Write linear functions to model (a) cost, (b) revenue, and (c) profit.

(a) Since the cost function is linear, it will have the form $C(x) = mx + b$ with $m = 120$ and $b = 2400$.

$$C(x) = 120x + 2400$$

(b) The revenue function is $R(x) = px$ with $p = 150$.

$$R(x) = 150x$$

2.4 Example 9 Writing Linear Cost, Revenue, and Profit Functions (cont.)

(c) The profit is the difference between the revenue and the cost.

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 150x - (120x + 2400) \\ &= 30x - 2400 \end{aligned}$$

(d) How many items must be sold for the company to make a profit?

To make a profit, $P(x)$ must be positive.

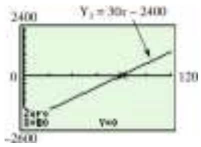
$$30x - 2400 > 0 \Rightarrow 30x > 2400 \Rightarrow x > 80$$

The company must sell at least 81 items to make a profit.

2.4 **Example 9** Writing Linear Cost, Revenue, and Profit Functions (cont.)

Graphing calculator solution

Define Y_1 as $30x - 2400$ and graph the line.
Then find the x-intercept.



The graph shows that y-values for x less than 80 are negative, y-values for x greater than 80 are positive. So, at least 81 items must be sold for the company to make a profit.