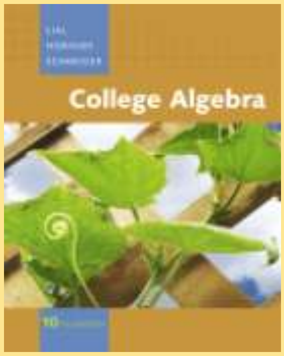
2

Graphs and Functions

Sections 2.5–2.8



2 Graphs and Functions

2.5 Equations of Lines; Curve Fitting

Point-Slope Form Slope-Intercept Form Vertical and Horizontal Lines Parallel and Perpendicular Lines Modeling Data Modeling Data Solving Linear Equations in One Variable by Graphing

2.5 Example 1 Using the Point-Slope Form (Given a Point and the Slope) (page 232)

Find an equation of the line through (3, –5) having slope –2.

Point-slope form: $y - y_1 = m(x - x_1)$

$x_1 = 3$, $y_1 = -5$, $m = -2$

$$y - (-5) = -2(x - 3)$$
$$y + 5 = -2x + 6$$
$$y = -2x + 1$$

2.5 Example 2 Using the Point-Slope Form (Given Two Points) (page 233)

Find an equation of the line through (–4, 3) and (5, –1).

First, find the slope: $m = \frac{3-(-1)}{-4-5} = -\frac{4}{9}$

Use either point for (x_1, y_1)

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - 3 = -\tfrac{4}{9}[x - (-4)] \quad (x_1, y_1) = (-4, 3)$$
$$9(y - 3) = -4(x + 4)$$
$$9y - 27 = -4x - 16$$
$$9y = -4x + 11$$
$$y = -\tfrac{4}{9}x + \tfrac{11}{9}$$

2.5 Example 2 Using the Point-Slope Form (Given Two Points) (page 233)

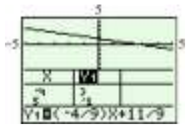
Find an equation of the line through (–4, 3) and (5, –1).

Verify using (5, –1) for (x_1, y_1) :

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - (-1) = -\tfrac{4}{9}[x - 5]$$
$$9(y + 1) = -4(x - 5)$$
$$9y + 9 = -4x + 20$$
$$9y = -4x + 11$$
$$y = -\tfrac{4}{9}x + \tfrac{11}{9}$$

2.5 Example 2 Using the Point-Slope Form (Given Two Points) (cont.)

The screen supports the result.



2.5 Example 3 Find the Slope and *y*-intercept From an Equation of a Line (page 234)

Find the slope and *y*-intercept of the line with equation $3x - 4y = 12$.

Write the equation in slope-intercept form:

$$3x - 4y = 12$$
$$-4y = -3x + 12$$
$$y = \frac{3}{4}x - 3$$

The slope is $\frac{3}{4}$ and the *y*-intercept is –3.

2.5 Example 4 Using the Slope-Intercept Form (Given Two Points) (page 234)

Find an equation of the line through (–2, 4) and (2, 2). Then graph the line using the slope-intercept form.

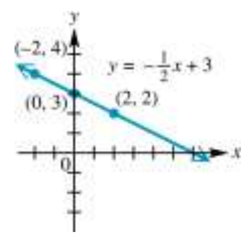
First, find the slope: $m = \frac{4-2}{-2-2} = -\frac{1}{2}$

Substitute $-\frac{1}{2}$ for *m* and the coordinates of one of the points (say, (2, 2)) for *x* and *y* into the slope-intercept form $y = mx + b$, then solve for *b*:

$$2 = -\frac{1}{2} \cdot 2 + b \Rightarrow 3 = b$$

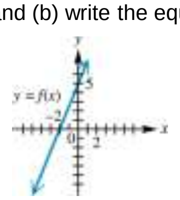
The equation is $y = -\frac{1}{2}x + 3$

2.5 Example 4 Using the Slope-Intercept Form (Given Two Points) (cont.)



2.5 Example 5 Finding an Equation From a Graph (page 235)

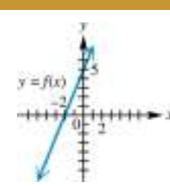
Use the graph to (a) find the slope, *y*-intercept, and *x*-intercept, and (b) write the equation of the function.



The line rises 5 units each time the *x*-value increases by 2 units.

The slope is $\frac{5}{2}$.

2.5 Example 5(a) Finding an Equation From a Graph (cont.)

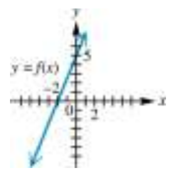


The graph intersects the *y*-axis at (5, 0) and the *x*-axis at (–2, 0).

The *y*-intercept is 5.

The *x*-intercept is –2.

2.5 **Example 5(b)** Finding an Equation From a Graph (cont.)



Slope $\frac{5}{2}$, *y*-intercept 5

$f(x) = \frac{5}{2}x + 5$

2.5 **Example 6(a)** Finding Equations of Parallel and Perpendicular Lines (page 236)

Find the equation in slope-intercept form of the line that passes through the point (2, –4) that is parallel to the line 3*x* – 2*y* = 5.

Write the equation in slope-intercept form to find the slope:

$$3x - 2y = 5 \Rightarrow -2y = -3x + 5 \Rightarrow y = \frac{3}{2}x + \frac{5}{2}$$

The slope is $\frac{3}{2}$.

Parallel lines have the same slope, so the slope of the line whose equation is to be found is $\frac{3}{2}$.

2.5 **Example 6(a)** Finding Equations of Parallel and Perpendicular Lines (page 236)

Find the equation in slope-intercept form of the line that passes through the point (2, –4) that is parallel to the line 3*x* – 2*y* = 5.

$y - y_1 = m(x - x_1)$ Point-slope form

$$y - (-4) = \frac{3}{2}(x - 2)$$

$$y + 4 = \frac{3}{2}x - 3$$

$$y = \frac{3}{2}x - 7$$

2.5 **Example 6(b)** Finding Equations of Parallel and Perpendicular Lines (page 236)

Find the equation in slope-intercept form of the line that passes through the point (2, –4) that is perpendicular to the line 3*x* – 2*y* = 5.

Write the equation in slope-intercept form to find the slope:

$$3x - 2y = 5 \Rightarrow -2y = -3x + 5 \Rightarrow y = \frac{3}{2}x + \frac{5}{2}$$

The slope is $\frac{3}{2}$.

The slopes of perpendicular lines are negative reciprocals, so the slope of the line whose equation is to be found is $-\frac{2}{3}$.

2.5 **Example 6(b)** Finding Equations of Parallel and Perpendicular Lines (page 236)

Find the equation in slope-intercept form of the line that passes through the point (2, –4) that is perpendicular to the line 3*x* – 2*y* = 5.

$y - y_1 = m(x - x_1)$ Point-slope form

$$y - (-4) = -\frac{2}{3}(x - 2)$$

$$y + 4 = -\frac{2}{3}x + \frac{4}{3}$$

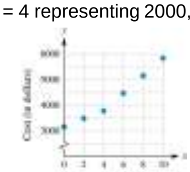
$$y = -\frac{2}{3}x - \frac{8}{3}$$

2.5 **Example 7** Finding an Equation of a Line That Models Data (page 238)

Average annual tuition and fees for in-state students at public 4-year colleges are shown in the table for selected years and in the graph below, with *x* = 0 representing 1996, *x* = 4 representing 2000, etc.

Year	Cost (in dollars)
1996	3151
1998	3486
2000	3774
2002	4461
2004	5148
2006	5836

Source: U.S. National Center for Education Statistics; College Board.



2.5 **Example 7(a)** Finding an Equation of a Line That Models Data (page 238)

Find an equation that models the data. Use the data for 1998 and 2004.

Year	Cost (in dollars)
1996	3151
1998	3486
2000	3774
2002	4461
2004	5148
2006	5836

Source: U.S. National Center for Education Statistics; College Board.

1998 is represented by $x = 2$ and 2004 is represented by $x = 8$.

Find the slope:

$$m = \frac{5148 - 3486}{8 - 2} = \frac{1662}{6} = 277$$

2.5 **Example 7(a)** Finding an Equation of a Line That Models Data (cont.)

Use either point, (2, 3486) or (8, 5148) for (x_1, y_1)

$$y - y_1 = m(x - x_1)$$
$$y - 3486 = 277(x - 2) \quad (x_1, y_1) = (2, 3486)$$
$$y - 3486 = 277x - 554$$
$$y = 277x + 2932$$

2.5 **Example 7(b)** Finding an Equation of a Line That Models Data (page 238)

Use the equation from part (a) to predict the cost of tuition and fees in 2008.

For 2008, $x = 12$.

$$y = 277x + 2932$$
$$y = 277(12) + 2932 = 6256$$

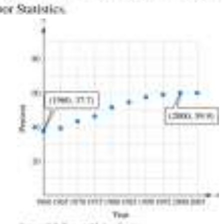
According to the model, average tuition and fees will be $6256 in 2008.

2.5 **Example 8** Finding an Equation of a Line That Models Data (page 239)

The table and graph illustrate how the percent of women in the civilian labor force has changed from 1960 to 2005.

Year	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005
% Women	37.7	39.3	43.3	46.3	51.5	54.5	57.5	58.9	59.9	59.3

Source: U.S. Bureau of Labor Statistics.



2.5 **Example 8(a)** Finding an Equation of a Line That Models Data (page 239)

Use the points (1965, 39.3) and (1995, 58.9) to find a linear equation that models the data.

Find the slope: $m = \frac{58.9 - 39.3}{1995 - 1965} = \frac{19.6}{30}$

Use either point for (x_1, y_1).

$$y - y_1 = m(x - x_1)$$
$$y - 58.9 = \frac{19.6}{30}(x - 1995) \quad (x_1, y_1) = (1995, 58.9)$$
$$y - 58.9 = .6533x - 1303.4$$
$$y = .6533x - 1244.5$$

2.5 **Example 8(b)** Finding an Equation of a Line That Models Data (page 239)

Use the equation to estimate the percent for 2005. How does the result compare to the actual figure of 59.3%?

Let $x = 2005$. Solve for y:

$$y = .6533x - 1244.5$$
$$y = .6533(2005) - 1244.5 \approx 65.4$$

The model estimates about 65.4% in 2005.

This is 6.1% more than the actual figure of 59.3%.

2.5 **Example 9** Solving an Equation with a Graphing Calculator (page 241)

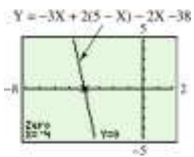
Use a graphing calculator to solve

$-3x + 2(5 - x) = 2x + 28$

Write the equation as an equivalent equation with 0 on one side:

$-3x + 2(5 - x) = 2x + 38 \Rightarrow -3x + 2(5 - x) - 2x - 38 = 0$

Graph $Y = -3x + 2(5 - x) - 2x - 38$, then find the x-intercept.



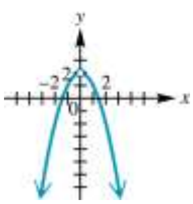
The solution set is {–4}.

2.6 Graphs of Basic Functions

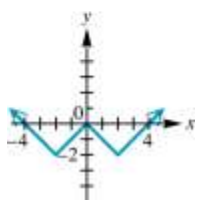
Continuity The Identity, Squaring, and Cubing Functions
The Square Root and Cube Root Functions
The Absolute Value Function Piecewise-Defined Functions
The Relation $x = y^2$

2.6 **Example 1** Determining Intervals of Continuity (page 248)

Describe the intervals of continuity for each function.



$(-\infty, 0)$ and $(0, \infty)$



$(-\infty, \infty)$

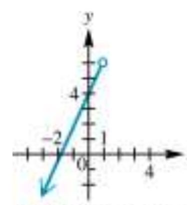
2.6 **Example 2(a)** Graphing Piecewise-Defined Functions (page 251)

Graph $f(x) = \begin{cases} 2x + 4 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$

Graph each interval of the domain separately.

If $x < 1$, the graph of $f(x) = 2x + 4$ has an endpoint at (1, 6), which is ***not*** included as part of the graph.

To find another point on this part of the graph, choose $x = 0$, so $y = 4$. Draw the ray starting at (1, 6) and extending through (0, 4).



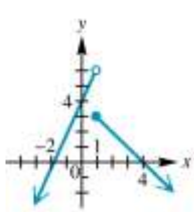
$f(x) = 2x + 4 \quad \text{if } x < 1$

2.6 **Example 2(a)** Graphing Piecewise-Defined Functions (cont.)

Graph $f(x) = \begin{cases} 2x + 4 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$

If $x \geq 1$, $f(x) = 4 - x$ has an endpoint at (1, 3), which is included as part of the graph.

Find another point, say (4, 0), and draw the ray starting at (1, 3) which extends through (4, 0).

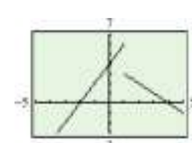


$f(x) = \begin{cases} 2x + 4 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$

2.6 **Example 2(a)** Graphing Piecewise-Defined Functions (cont.)

Graph $f(x) = \begin{cases} 2x + 4 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$

Graphing calculator solution



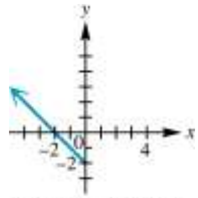
2.6 **Example 2(b)** Graphing Piecewise-Defined Functions (page 251)

Graph $f(x)=\begin{cases}-x-2 & \text{if } x\le 0\\ \frac{1}{2}x-2 & \text{if } x>0\end{cases}$

Graph each interval of the domain separately.

If $x \le 0$, the graph of $f(x) = -x - 2$ has an endpoint at (0, −2), which is included as part of the graph.

To find another point on this part of the graph, choose $x = -2$, so $y = 0$. Draw the ray starting at (0, −2) and extending through (−2, 0).



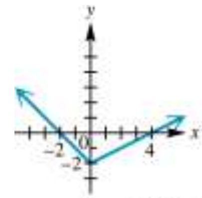
2.6 **Example 2(b)** Graphing Piecewise-Defined Functions (cont.)

Graph $f(x)=\begin{cases}-x-2 & \text{if } x\le 0\\ \frac{1}{2}x-2 & \text{if } x>0\end{cases}$

If $x > 0$, the graph of $f(x)=\frac{1}{2}x-2$ has an endpoint at (0, −2), which is ***not*** included as part of the graph.

Find another point, say (4, 0), and draw the ray starting at (0, −2) which extends through (4, 0).

Note that the two endpoints coincide, so (0, −2) is included as part of the graph.

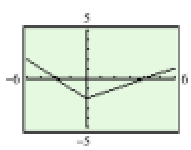


$f(x)=\begin{cases}-x-2 & \text{if } x\le 0\\ \frac{1}{2}x-2 & \text{if } x>0\end{cases}$

2.6 **Example 2(b)** Graphing Piecewise-Defined Functions (cont.)

Graph $f(x)=\begin{cases}-x-2 & \text{if } x\le 0\\ \frac{1}{2}x-2 & \text{if } x>0\end{cases}$

Graphing calculator solution



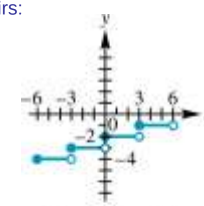
2.6 **Example 3** Graphing a Greatest Integer Function (page 253)

Graph $f(x)=\left[\!\left[\frac{1}{3}x-2\right]\!\right]$.

Create a table of sample ordered pairs:

x	−6	−3	$-\frac{3}{2}$	0
$\left[\!\left[\frac{1}{3}x-2\right]\!\right]$	−4	−3	−3	−2

x	$\frac{3}{2}$	3	6
$\left[\!\left[\frac{1}{3}x-2\right]\!\right]$	−2	−1	0



$f(x)=\left[\!\left[\frac{1}{3}x-2\right]\!\right]$

2.6 **Example 4** Applying a Greatest Integer Function (page 254)

An express mail company charges \$20 for a package weighing up to 2 lb and \$2 for each additional pound or fraction of a pound. Let $y = C(x)$ represent the cost to send a package weighing x pounds. Graph $y = C(x)$ for x in the interval (0, 6].

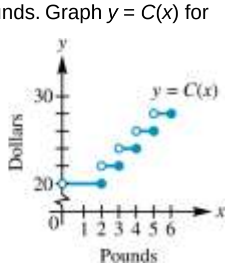
For x in (0, 2], $y = 20$.

For x in (2, 3], $y = 20 + 2 = 22$.

For x in (3, 4], $y = 22 + 2 = 24$.

For x in (4, 5], $y = 24 + 2 = 26$.

For x in (5, 6], $y = 26 + 2 = 28$.



2.7 Graphing Techniques

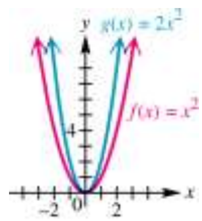
Stretching and Shrinking Reflecting Symmetry
Even and Odd Functions Translations

2.7 **Example 1(a)** Stretching or Shrinking a Graph (page 259)

Graph the function $g(x) = 2x^2$.

Create a table of values.

x	$f(x) = x^2$	$g(x) = 2x^2$
–2	4	8
–1	1	2
0	0	0
1	1	2
2	4	8



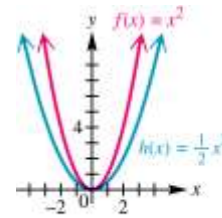
Note that for corresponding values of *x*, the *y*-values of *g*(*x*) are each twice that of *f*(*x*).

2.7 **Example 1(b)** Stretching or Shrinking a Graph (page 259)

Graph the function $h(x) = \frac{1}{2}x^2$.

Create a table of values.

x	$f(x) = x^2$	$h(x) = \frac{1}{2}x^2$
–2	4	2
–1	1	½
0	0	0
1	1	½
2	4	2



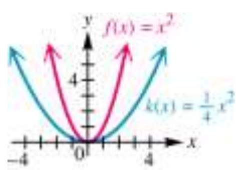
Note that for corresponding values of *x*, the *y*-values of *g*(*x*) are each half that of *f*(*x*).

2.7 **Example 1(c)** Stretching or Shrinking a Graph (page 259)

Graph the function $k(x) = \frac{1}{4}x^2$.

Create a table of values.

x	$f(x) = x^2$	$k(x) = \frac{1}{4}x^2$
–2	4	1
–1	1	¼
0	0	0
1	1	¼
2	4	1



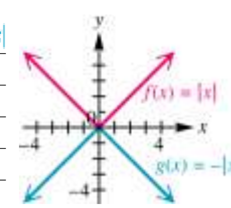
Note that for corresponding values of *x*, the *y*-values of *g*(*x*) are each one-fourth that of *f*(*x*).

2.7 **Example 2(a)** Reflecting a Graph Across an Axis (page 261)

Graph the function $g(x) = -|x|$

Create a table of values.

x	$f(x) = \lvert x\rvert$	$g(x) = -\lvert x\rvert$
–2	2	–2
–1	1	–1
0	0	0
1	1	–1
2	2	–2



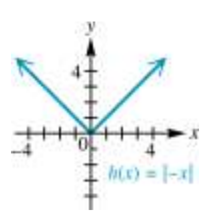
Note that every *y*-value of *g*(*x*) is the negative of the corresponding *y*-value of *f*(*x*). The graph of *f*(*x*) is reflected across the *x*-axis to give the graph of *g*(*x*).

2.7 **Example 2(b)** Reflecting a Graph Across an Axis (page 261)

Graph the function $h(x) = |-x|$

Create a table of values.

x	$f(x) = \lvert x\rvert$	$h(x) = \lvert -x\rvert$
–2	2	2
–1	1	1
0	0	0
1	1	1
2	2	2



Note that every *y*-value of *g*(*x*) is the same of the corresponding *y*-value of *f*(*x*). The graph of *f*(*x*) is reflected across the *y*-axis to give the graph of *g*(*x*).

2.7 **Example 3(a)** Testing for Symmetry with Respect to an Axis (page 262)

Test $x = |y|$ for symmetry with respect to the *x*-axis and the *y*-axis.

Replace *x* with –*x*: $-x = |y|$

The result is not the same as the original equation.

The graph is not symmetric with respect to the *y*-axis.

Replace *y* with –*y*: $x = |-y| \Rightarrow x = |y|$

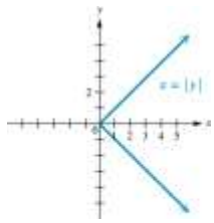
The result is the same as the original equation.

The graph is symmetric with respect to the *x*-axis.

The graph is symmetric with respect to the *x*-axis only.

2.7 Example 3(a) Testing for Symmetry with Respect to an Axis (cont.)

Test $x = |y|$ for symmetry with respect to the *x*-axis and the *y*-axis.



2.7 Example 3(b) Testing for Symmetry with Respect to an Axis (page 262)

Test $y = |x| - 3$ for symmetry with respect to the *x*-axis and the *y*-axis.

Replace *x* with –*x*: $y = |-x| - 3 \Rightarrow y = |x| - 3$

The result is the same as the original equation.

The graph is symmetric with respect to the *y*-axis.

Replace *y* with –*y*: $-y = |x| - 3$

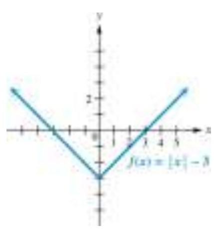
The result is the not same as the original equation.

The graph is not symmetric with respect to the *x*-axis.

The graph is symmetric with respect to the *y*-axis only.

2.7 Example 3(b) Testing for Symmetry with Respect to an Axis (cont.)

Test $y = |x| - 3$ for symmetry with respect to the *x*-axis and the *y*-axis.



2.7 Example 3(c) Testing for Symmetry with Respect to an Axis (page 262)

Test $2x - y = 6$ for symmetry with respect to the *x*-axis and the *y*-axis.

Replace *x* with –*x*: $2(-x) - y = 6 \Rightarrow -2x - y = 6$

The result is not the same as the original equation.

The graph is not symmetric with respect to the *y*-axis.

Replace *y* with –*y*: $2x - (-y) = 6 \Rightarrow 2x + y = 6$

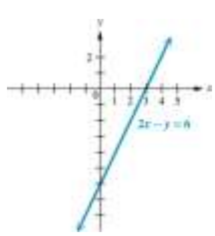
The result is the not same as the original equation.

The graph is not symmetric with respect to the *x*-axis.

The graph is not symmetric with respect to either axis.

2.7 Example 3(c) Testing for Symmetry with Respect to an Axis (cont.)

Test $2x - y = 6$ for symmetry with respect to the *x*-axis and the *y*-axis.



2.7 Example 3(d) Testing for Symmetry with Respect to an Axis (page 262)

Test $x^2 + y^2 = 25$ for symmetry with respect to the *x*-axis and the *y*-axis.

Replace *x* with –*x*: $(-x)^2 + y^2 = 25 \Rightarrow x^2 + y^2 = 25$

The result is the same as the original equation.

The graph is symmetric with respect to the *y*-axis.

Replace *y* with –*y*: $x^2 + (-y)^2 = 25 \Rightarrow x^2 + y^2 = 25$

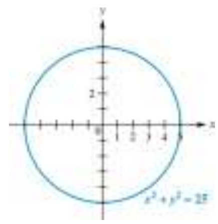
The result is same as the original equation.

The graph is symmetric with respect to the *x*-axis.

The graph is symmetric with respect to both axes.

2.7 **Example 3(d)** Testing for Symmetry with Respect to an Axis (cont.)

Test $x^2 + y^2 = 25$ for symmetry with respect to the x-axis and the y-axis.



2.7 **Example 4(a)** Testing for Symmetry with Respect to the Origin (page 264)

Is the graph of $y = -2x^3$ symmetric with respect to the origin?

Replace x with $-x$ and y with $-y$:

$$-y = -2(-x)^3 \Rightarrow -y = 2x^3 \Rightarrow y = -2x^3$$

The result is the same as the original equation.

The graph is symmetric with respect to the origin.

2.7 **Example 4(b)** Testing for Symmetry with Respect to the Origin (page 264)

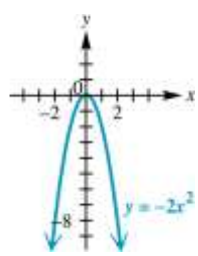
Is the graph of $y = -2x^2$ symmetric with respect to the origin?

Replace x with $-x$ and y with $-y$:

$$-y = -2(-x)^2 \Rightarrow -y = -2x^2 \Rightarrow y = 2x^2$$

The result is not the same as the original equation.

The graph is not symmetric with respect to the origin.



2.7 **Example 5(a)** Determining Whether Functions are Even, Odd, or Neither (page 265)

Is the function even, odd, or neither?

$$g(x) = x^5 + 2x^3 - 3x$$

Replace x with $-x$:

$$\begin{aligned} g(-x) &= (-x)^5 + 2(-x)^3 - 3(-x) \\ &= -x^5 - 2x^3 + 3x \\ &= -(x^5 + 2x^3 - 3x) \\ &= -g(x) \end{aligned}$$

$g(x)$ is an odd function.

2.7 **Example 5(b)** Determining Whether Functions are Even, Odd, or Neither (page 265)

Is the function even, odd, or neither?

$$h(x) = 2x^2 - 3$$

Replace x with $-x$:

$$\begin{aligned} h(-x) &= 2(-x)^2 - 3 \\ &= 2x^2 - 3 \\ &= h(x) \end{aligned}$$

$h(x)$ is an even function.

2.7 **Example 5(c)** Determining Whether Functions are Even, Odd, or Neither (page 265)

Is the function even, odd, or neither?

$$k(x) = x^2 + 6x + 9$$

Replace x with $-x$:

$$\begin{aligned} k(-x) &= (-x)^2 + 6(-x) + 9 \\ &= x^2 - 6x + 9 \end{aligned}$$

$$k(-x) \neq k(x) \text{ and } k(-x) \neq -k(x)$$

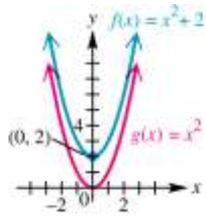
$k(x)$ is neither even nor odd.

2.7 Example 6 Translating a Graph Vertically (page 261)

Graph $f(x) = x^2 + 2$.

Compare a table of values for $g(x) = x^2$ with $f(x) = x^2 + 2$.

x	$g(x) = x^2$	$f(x) = x^2 + 2$
–2	4	6
–1	1	3
0	0	2
1	1	3
2	4	6



The graph of $f(x)$ is the same as the graph of $g(x)$ translated 2 units up.

2.7 Example 7 Translating a Graph Horizontally (page 266)

Graph $f(x) = (x + 2)^2$.

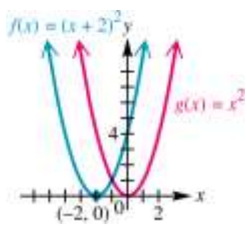
Compare a table of values for $g(x) = x^2$ with $f(x) = (x + 2)^2$.

x	$g(x) = x^2$	$f(x) = (x + 2)^2$
–4	16	4
–3	9	1
–2	4	0
–1	1	1
0	0	4
1	1	9

2.7 Example 7 Translating a Graph Horizontally (cont.)

Graph $f(x) = (x + 2)^2$.

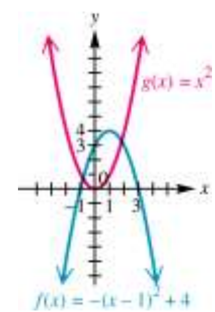
The graph of $f(x)$ is the same as the graph of $g(x)$ translated 2 units left.



2.7 Example 8(a) Using More Than One Transformation on Graphs (page 268)

Graph $f(x) = -(x - 1)^2 + 4$.

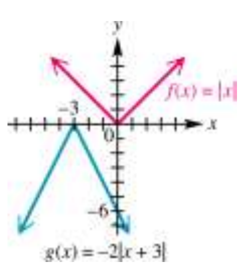
This is the graph of $g(x) = x^2$ translated one unit to the right, reflected across the x-axis, and then translated four units up.



2.7 Example 8(b) Using More Than One Transformation on Graphs (page 268)

Graph $f(x) = -2|x + 3|$.

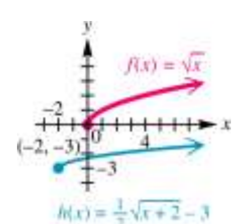
This is the graph of $g(x) = |x|$ translated three units to the left, reflected across the x-axis, and then stretched vertically by a factor of 2.



2.7 Example 8(c) Using More Than One Transformation on Graphs (page 268)

Graph $h(x) = \frac{1}{2}\sqrt{x + 2} - 3$.

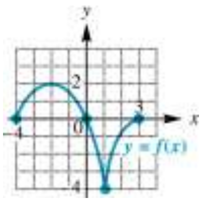
This is the graph of $g(x) = \sqrt{x}$ translated two units to the left, shrunk vertically by a factor of 2, then translated 3 units down.

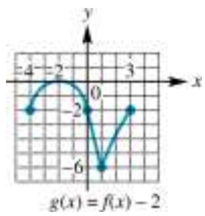


2.7 **Example 9(a)** Graphing Translations Given the Graph of $y = f(x)$ (page 269)

Use the graph of $f(x)$ to sketch the graph of
$g(x) = f(x) – 2$.

Translate the graph of $f(x)$ 2 units down.

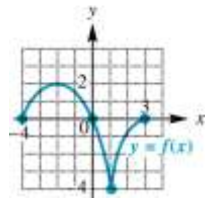


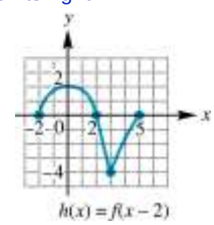


2.7 **Example 9(b)** Graphing Translations Given the Graph of $y = f(x)$ (page 269)

Use the graph of $f(x)$ to sketch the graph of
$h(x) = f(x – 2)$.

Translate the graph of $f(x)$ 2 units right.

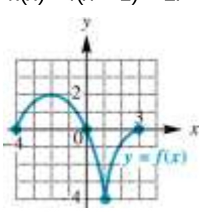


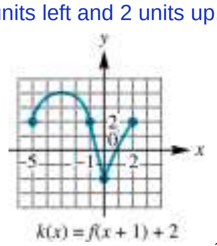


2.7 **Example 9(c)** Graphing Translations Given the Graph of $y = f(x)$ (page 269)

Use the graph of $f(x)$ to sketch the graph of
$k(x) = f(x + 1) + 2$.

Translate the graph of $f(x)$ 1 units left and 2 units up.





2.8 Function Operations and Composition

Arithmetic Operations on Functions   The Difference Quotient
Composition of Functions and Domain

2.8 **Example 1** Using Operations on Functions (page 275)

Let $f(x) = 3x – 4$ and $g(x) = 2x^2 – 1$. Find:

(a) $(f + g)(0)$

$f(0) = 3(0) – 4 = –4$ and $g(0) = 2(0)^2 – 1 = –1$, so
$(f + g)(0) = –4 – 1 = –5$.

(b) $(f – g)(4)$

$f(4) = 3(4) – 4 = 8$ and $g(4) = 2(4)^2 – 1 = 31$, so
$(f – g)(4) = 8 – 31 = –23$.

2.8 **Example 1** Using Operations on Functions (cont.)

Let $f(x) = 3x – 4$ and $g(x) = 2x^2 – 1$. Find:

(c) $(fg)(–2)$

$f(–2) = 3(–2) – 4 = –10$ and
$g(–2) = 2(–2)^2 – 1 = 7$, so
$(fg)(–2) = (–10)(7) = –70$.

(d) $\left(\frac{f}{g}\right)(3)$

$f(3) = 3(3) – 4 = 5$ and $g(3) = 2(3)^2 – 1 = 17$, so
$\left(\frac{f}{g}\right)(3) = \frac{5}{17}$.

2.8 **Example 2(a)** Using Operations on Functions (page 276)

Let $f(x) = x^2 - 3x$ and $g(x) = 4x + 5$. Find $(f + g)(x)$ and give the domain.

$(f + g)(x) = (x^2 - 3x) + (4x + 5) = x^2 + x + 5$

The domain of $f + g$ is the intersection of the domains of f and g.

Domain of f: $(-\infty, \infty)$ Domain of g: $(-\infty, \infty)$

The domain of $f + g$ is $(-\infty, \infty)$

2.8 **Example 2(b)** Using Operations on Functions (page 276)

Let $f(x) = x^2 - 3x$ and $g(x) = 4x + 5$. Find $(f - g)(x)$ and give the domain.

$(f - g)(x) = (x^2 - 3x) - (4x + 5) = x^2 - 7x - 5$

The domain of $f - g$ is the intersection of the domains of f and g.

Domain of f: $(-\infty, \infty)$ Domain of g: $(-\infty, \infty)$

The domain of $f - g$ is $(-\infty, \infty)$

2.8 **Example 2(c)** Using Operations on Functions (page 276)

Let $f(x) = x^2 - 3x$ and $g(x) = 4x + 5$. Find $(fg)(x)$ and give the domain.

$(fg)(x) = (x^2 - 3x)(4x + 5) = 4x^3 + 5x^2 - 12x^2 - 15x$

$= 4x^3 - 7x^2 - 15x$

The domain of fg is the intersection of the domains of f and g.

Domain of f: $(-\infty, \infty)$ Domain of g: $(-\infty, \infty)$

The domain of fg is $(-\infty, \infty)$

2.8 **Example 2(d)** Using Operations on Functions (page 276)

Let $f(x) = x^2 - 3x$ and $g(x) = 4x + 5$. Find $\left(\frac{f}{g}\right)(x)$ and give the domain.

$$\left(\frac{f}{g}\right)(x) = \frac{x^2 - 3x}{4x + 5}$$

The domain of $\frac{f}{g}$ is the intersection of the domains of f and g such that $g(x) \neq 0$.

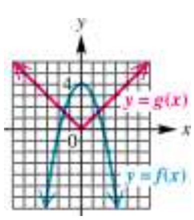
Domain of f: $(-\infty, \infty)$ Domain of g: $(-\infty, \infty)$

$g(x) = 0 \Rightarrow 4x + 5 = 0 \Rightarrow x = -\frac{5}{4}$

The domain of $\frac{f}{g}$ is $\left(-\infty, -\frac{5}{4}\right) \cup \left(-\frac{5}{4}, \infty\right)$

2.8 **Example 3(a)** Evaluating Combinations of Functions (page 277)

Use the representations of the functions f and g to evaluate $(f + g)(1)$, $(f - g)(0)$, $(fg)(-1)$, and $\left(\frac{f}{g}\right)(-2)$.

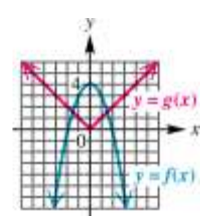


$f(1) = 3$ and $g(1) = 1$, so
$(f + g)(1) = 3 + 1 = 4$

$f(0) = 4$ and $g(0) = 0$, so
$(f - g)(1) = 4 - 0 = 4$

2.8 **Example 3(a)** Evaluating Combinations of Functions (cont.)

Use the representations of the functions f and g to evaluate $(f + g)(1)$, $(f - g)(0)$, $(fg)(-1)$, and $\left(\frac{f}{g}\right)(-2)$.



$f(-1) = 3$ and $g(-1) = 1$, so
$(fg)(-1) = (3)(1) = 3$

$f(-2) = 0$ and $g(-2) = 2$, so
$\left(\frac{f}{g}\right)(-2) = \frac{0}{2} = 0$

2.8 Example 3(b) Evaluating Combinations of Functions (page 277)

Use the representations of the functions *f* and *g* to evaluate $(f + g)(1)$, $(f - g)(0)$, $(fg)(-1)$, and $\left(\frac{f}{g}\right)(-2)$.

x	f(x)	g(x)
−2	−5	0
−1	−3	2
0	−1	4
1	1	6

$f(1) = 1$ and $g(1) = 6$, so $(f + g)(1) = 1 + 6 = 7$

$f(0) = -1$ and $g(0) = 4$, so $(f - g)(1) = -1 - 4 = -5$

2.8 Example 3(b) Evaluating Combinations of Functions (cont.)

Use the representations of the functions *f* and *g* to evaluate $(f + g)(1)$, $(f - g)(0)$, $(fg)(-1)$, and $\left(\frac{f}{g}\right)(-2)$.

x	f(x)	g(x)
−2	−5	0
−1	−3	2
0	−1	4
1	1	6

$f(-1) = -3$ and $g(-1) = 2$, so $(fg)(-1) = (-3)(2) = -6$

$f(-2) = -5$ and $g(0) = 0$, so $\left(\frac{f}{g}\right)(-2) = \frac{-5}{0}$, which is undefined

2.8 Example 3(c) Evaluating Combinations of Functions (page 277)

Use the representations of the functions *f* and *g* to evaluate $(f + g)(1)$, $(f - g)(0)$, $(fg)(-1)$, and $\left(\frac{f}{g}\right)(-2)$.

$$f(x) = 3x + 4,\ g(x) = -|x|$$

$f(1) = 3(1) + 4 = 7$ and $g(1) = -|1| = -1$, so
$(f + g)(1) = 7 - 1 = 6$

$f(0) = 3(0) + 4 = 4$ and $g(0) = -|0| = 0$, so
$(f - g)(0) = 4 - 0 = 4$

2.8 Example 3(c) Evaluating Combinations of Functions (page 277)

Use the representations of the functions *f* and *g* to evaluate $(f + g)(1)$, $(f - g)(0)$, $(fg)(-1)$, and $\left(\frac{f}{g}\right)(-2)$.

$$f(x) = 3x + 4,\ g(x) = -|x|$$

$f(-1) = 3(-1) + 4 = 1$ and $g(-1) = -|-1| = -1$, so
$(fg)(1) = (1)(-1) = -1$

$f(-2) = 3(-2) + 4 = -2$ and $g(-2) = -|-2| = -2$, so
$\left(\frac{f}{g}\right)(-2) = \frac{-2}{-2} = 1$

2.8 Example 4 Find the Difference Quotient (page 278)

Let $f(x) = 3x^2 - 2x + 4$. Find the difference quotient and simplify the expression.

Step 1 Find $f(x + h)$

$$\begin{aligned} f(x + h) &= 3(x + h)^2 - 2(x + h) + 4 \\ &= 3(x^2 + 2xh + h^2) - 2x - 2h + 4 \\ &= 3x^2 + 6xh + 3h^2 - 2x - 2h + 4 \end{aligned}$$

2.8 Example 4 Find the Difference Quotient (cont.)

Let $f(x) = 3x^2 - 2x + 4$. Find the difference quotient and simplify the expression.

Step 2 Find $f(x + h) - f(x)$

$$\begin{aligned} &f(x + h) - f(x) \\ &= (3x^2 + 6xh + 3h^2 - 2x - 2h + 4) - (3x^2 - 2x + 4) \\ &= 3x^2 + 6xh + 3h^2 - 2x - 2h + 4 - 3x^2 + 2x - 4 \\ &= 6xh + 3h^2 - 2h \end{aligned}$$

2.8 Example 4 Find the Difference Quotient (cont.)

Let $f(x) = 3x^2 - 2x + 4$. Find the difference quotient and simplify the expression.

Step 3 Find the difference quotient, $\frac{f(x+h) - f(x)}{h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{6xh + 3h^2 - 2h}{h} = 6x + 3h - 2$$

2.8 Example 5(a) Evaluating Composite Functions (page 279)

Let $f(x) = \sqrt{x+4}$ and $g(x) = \frac{2}{x}$. Find $(f \circ g)(2)$.

First find $g(2)$: $g(2) = \frac{2}{2} = 1$

Now find $(f \circ g)(2)$:

$$(f \circ g)(2) = f(g(2)) = f(1) = \sqrt{1+4} = \sqrt{5}$$

2.8 Example 5(b) Evaluating Composite Functions (page 279)

Let $f(x) = \sqrt{x+4}$ and $g(x) = \frac{2}{x}$. Find $(g \circ f)(5)$.

First find $f(5)$: $f(5) = \sqrt{5+4} = \sqrt{9} = 3$

Now find $(g \circ f)(5)$:

$$(g \circ f)(5) = g(f(5)) = g(3) = \frac{2}{3}$$

2.8 Example 6(a) Determining Composite Functions and Their Domains (page 280)

Let $f(x) = \sqrt{x-1}$ and $g(x) = 2x + 5$. Find $(f \circ g)(x)$ and determine its domain.

$$(f \circ g)(x) = f(g(x)) = \sqrt{(2x+5) - 1} = \sqrt{2x+4}$$

Domain and range of g: $(-\infty, \infty)$

Domain of f: $[1, \infty)$

Therefore, $g(x)$ must be greater than 1:

$$2x + 5 \geq 1 \Rightarrow x \geq -2$$

Domain of $(f \circ g)(x)$: $[-2, \infty)$

2.8 Example 6(b) Determining Composite Functions and Their Domains (page 280)

Let $f(x) = \sqrt{x-1}$ and $g(x) = 2x + 5$. Find $(g \circ f)(x)$ and determine its domain.

$$(g \circ f)(x) = g(f(x)) = 2\sqrt{x-1} + 5$$

Domain of f: $[1, \infty)$ Range of f: $[0, \infty)$

Domain of g: $(-\infty, \infty)$

Therefore, the domain of $(f \circ g)(x)$ is portion of the domain of g that intersects with the domain of f.

Domain of $(g \circ f)(x)$: $[1, \infty)$

2.8 Example 7(a) Determining Composite Functions and Their Domains (page 280)

Let $f(x) = \frac{5}{x+4}$ and $g(x) = \frac{2}{x}$. Find $(f \circ g)(x)$ and determine its domain.

$$(f \circ g)(x) = f(g(x)) = \frac{5}{(2/x) + 4} = \frac{5x}{2 + 4x}$$

Domain and range of g: $(-\infty, 0) \cup (0, \infty)$

Domain of f: $(-\infty, -4) \cup (-4, \infty)$

Therefore, $g(x) \neq -4$: $\frac{2}{x} \neq -4 \Rightarrow x \neq -\frac{1}{2}$

Domain of $(f \circ g)(x)$: $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 0\right) \cup (0, \infty)$

2.8 **Example 7(b)** Determining Composite Functions and Their Domains (page 280)

Let $f(x)=\frac{5}{x+4}$ and $g(x)=\frac{2}{x}$. Find $(g\circ f)(x)$ and determine its domain.

$$(g\circ f)(x)=g(f(x))=\frac{2}{5/(x+4)}=\frac{2x+8}{5}$$

Domain of *f*: $(-\infty,-4)\cup(-4,\infty)$

Range of *f*: $(-\infty,0)\cup(0,\infty)$

Domain of *g*: $(-\infty,0)\cup(0,\infty)$

Since 0 is not in the range of *f*, the domain of $(g\circ f)(x)$ is $(-\infty,-4)\cup(-4,\infty)$

2.8 **Example 8** Showing that $(g\circ f)(x)\neq(f\circ g)(x)$ (page 281)

Let $f(x) = 2x – 5$ and $g(x) = 3x^2 + x$. Show that $(g\circ f)(x)\neq(f\circ g)(x)$ in general.

$$\begin{aligned}(g\circ f)(x)&=g(2x-5)=3(2x-5)^2+(2x-5)\\&=3(4x^2-20x+25)+2x-5\\&=12x^2-58x+70\end{aligned}$$

$$\begin{aligned}(f\circ g)(x)&=f(3x^2+x)=2(3x^2+x)-5\\&=6x^2+2x-5\end{aligned}$$

In general, $12x^2-58x+70\neq 6x^2+2x-5$

So, $(g\circ f)(x)\neq(f\circ g)(x)$

2.8 **Example 9** Finding Functions That Form a Given Composite (page 282)

Find functions *f* and *g* such that $(f\circ g)(x) = 4(3x + 2)^2 – 5(3x + 2) – 8$.

Note the repeated quantity $3x + 2$.

Choose $g(x) = 3x + 2$ and $f(x) = 4x^2 – 5x – 8$.

Then $(f\circ g)(x) = 4(3x + 2)^2 – 5(3x + 2) – 8$.

There are other pairs of functions *f* and *g* that also work. For instance, let $f(x) = 4(x + 2)^2 – 5(x + 2) – 8$ and $g(x) = 3x$.