



R Review of Basic Concepts

R.5 Rational Expressions
R.6 Rational Exponents
R.7 Radical Expressions

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R.5 Rational Expressions

Rational Expressions • Lowest Terms of a Rational Expression • Multiplication and Division • Addition and Subtraction • Complex Fractions

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R.5 Example 1(a) Writing Rational Expressions in Lowest Terms (page 44)

Write the rational expression in lowest terms.

$$(a) \frac{12q^2 - 30q}{4q^2 - 25} = \frac{6q(2q - 5)}{(2q + 5)(2q - 5)} \text{ Factor.}$$

$$= \frac{6q}{2q + 5} \text{ Divide out the common factor.}$$

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R.5 Example 1(b) Writing Rational Expressions in Lowest Terms (page 44)

Write the rational expression in lowest terms.

$$(b) \frac{m^2 - 8m + 16}{8m - 2m^2} = \frac{(m - 4)^2}{2m(4 - m)} \text{ Factor.}$$

$$= -\frac{(m - 4)^2}{2m(m - 4)} \text{ Multiply numerator and denominator by } -1.$$

$$= -\frac{m - 4}{2m} \text{ Divide out the common factor.}$$

$$= \frac{-(m - 4)}{2m} \text{ or } \frac{4 - m}{2m}$$

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R.5 Example 2(a) Multiplying or Dividing Rational Expressions (page 45)

Multiply.

$$\frac{6z^6}{7} \cdot \frac{28}{9z^2} = \frac{6z^6 \cdot 28}{7 \cdot 9z^2} \text{ Multiply.}$$

$$= \frac{2 \cdot 3 \cdot 4 \cdot 7 \cdot z^6 \cdot z^4}{7 \cdot 3 \cdot 3 \cdot z^2} \text{ Factor.}$$

$$= \frac{8z^4}{3} \text{ Divide out common factors, then simplify.}$$

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R.5 Example 2(b) Multiplying or Dividing Rational Expressions (page 45)

Multiply.

$$\begin{aligned} \frac{4n^2 + 3n - 10}{2n^2 + 3n - 2} \cdot \frac{2n - 1}{n + 4} &= \frac{(n+2)(4n-5)}{(n+2)(2n-1)} \cdot \frac{(2n-1)}{(n+4)} \quad \text{Factor.} \\ &= \frac{(n+2)(4n-5)(2n-1)}{(n+2)(2n-1)(n+4)} \quad \text{Multiply.} \\ &= \frac{4n-5}{n+4} \quad \text{Divide out common factors, then simplify.} \end{aligned}$$

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R.5 Example 2(c) Multiplying or Dividing Rational Expressions (page 45)

Divide.

$$\begin{aligned} \frac{5z^2 - 16z + 3}{z^2 + z - 12} \div \frac{30z^2 - 6z}{2z^3 + 8z^2} &= \frac{5z^2 - 16z + 3}{z^2 + z - 12} \cdot \frac{2z^3 + 8z^2}{30z^2 - 6z} \quad \text{Multiply by the reciprocal of the divisor.} \\ &= \frac{(5z-1)(z-3)}{(z+4)(z-3)} \cdot \frac{2z \cdot z(z+4)}{3 \cdot 2z(5z-1)} \quad \text{Factor.} \\ &= \frac{z}{3} \quad \text{Multiply, then divide out common factors.} \end{aligned}$$

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R.5 Example 2(d) Multiplying or Dividing Rational Expressions (page 45)

Multiply.

$$\begin{aligned} \frac{x^2 - 1}{x^3 - 1} \cdot \frac{xy - 2y + 3x - 6}{xy + 3x + y + 3} &= \frac{(x-1)(x+1)}{(x-1)(x^2 + x + 1)} \cdot \frac{y(x-2) + 3(x-2)}{x(y+3) + (y+3)} \quad \text{Factor.} \\ &= \frac{(x-1)(x+1)}{(x-1)(x^2 + x + 1)} \cdot \frac{(y+3)(x-2)}{(x+1)(y+3)} \\ &= \frac{x-2}{x^2 + x + 1} \quad \text{Multiply, then divide out common factors.} \end{aligned}$$

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R.5 Example 3(a) Adding or Subtracting Rational Expressions (page 47)

Add $\frac{3}{10z^4} + \frac{2}{15z^2}$

Find the LCD:

$$\left. \begin{aligned} 10z^4 &= 2 \cdot 5 \cdot z^4 \\ 15z^2 &= 3 \cdot 5 \cdot z^2 \end{aligned} \right\} \text{LCD} = 2 \cdot 3 \cdot 5 \cdot z^4 = 30z^4$$

$$\begin{aligned} \frac{3}{10z^4} + \frac{2}{15z^2} &= \frac{3 \cdot 3}{10z^4 \cdot 3} + \frac{2 \cdot 2z^2}{15z^2 \cdot 2z^2} \\ &= \frac{9}{30z^4} + \frac{4z^2}{30z^4} = \frac{9 + 4z^2}{30z^4} \end{aligned}$$

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R.5 Example 3(b) Adding or Subtracting Rational Expressions (page 47)

Add $\frac{7}{m-5} + \frac{2m}{5-m}$

Find the LCD: $\left. \begin{aligned} m-5 &= m-5 \\ 5-m &= (-1)(m-5) \end{aligned} \right\} \text{LCD} = m-5$

$$\begin{aligned} \frac{7}{m-5} + \frac{2m}{5-m} &= \frac{7}{m-5} + \frac{2m(-1)}{(5-m)(-1)} \\ &= \frac{7}{m-5} + \frac{-2m}{m-5} \\ &= \frac{7-2m}{m-5} \quad \text{or} \quad \frac{2m-7}{5-m} \end{aligned}$$

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R.5 Example 3(c) Adding or Subtracting Rational Expressions (page 47)

Subtract $\frac{4}{(x-3)(x+5)} - \frac{6}{(x+5)(x-5)}$

Find the LCD: $(x-3)(x+5)(x-5)$

$$\begin{aligned} \frac{4}{(x-3)(x+5)} - \frac{6}{(x+5)(x-5)} &= \frac{4(x-5)}{(x-3)(x+5)(x-5)} - \frac{6(x-3)}{(x-3)(x+5)(x-5)} \\ &= \frac{4x-20 - (6x-18)}{(x-3)(x+5)(x-5)} = \frac{-2x-2}{(x-3)(x+5)(x-5)} \end{aligned}$$

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R.5 Example 4(a) Simplifying Complex Fractions (page 49)

Simplify $\frac{3 + \frac{4}{x^2}}{6 - \frac{1}{x^2}}$

Multiply the numerator and denominator by the LCD of all the fractions, x^2 .

$$\frac{3 + \frac{4}{x^2}}{6 - \frac{1}{x^2}} = \frac{x^2 \left(3 + \frac{4}{x^2}\right)}{x^2 \left(6 - \frac{1}{x^2}\right)} = \frac{3x^2 + 4}{6x^2 - 1}$$

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R.5 Example 4(b) Simplifying Complex Fractions (page 49)

Simplify $\frac{\frac{1}{z+1} - \frac{1}{z-1}}{\frac{1}{z} + \frac{1}{z+1}}$

Multiply the numerator and denominator by the LCD of all the fractions, $z(z+1)(z-1)$.

$$\begin{aligned} \frac{\frac{1}{z+1} - \frac{1}{z-1}}{\frac{1}{z} + \frac{1}{z+1}} &= \frac{z(z+1)(z-1) \left(\frac{1}{z+1} - \frac{1}{z-1}\right)}{z(z+1)(z-1) \left(\frac{1}{z} + \frac{1}{z+1}\right)} \\ &= \frac{z(z-1) - z(z+1)}{(z+1)(z-1) + z(z-1)} \\ &= \frac{z^2 - z - z^2 - z}{z^2 - 1 + z^2 - z} = \frac{-2z}{2z^2 - z - 1} \end{aligned}$$

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R.6 Rational Exponents

Negative Exponents and the Quotient Rule •
Rational Exponents • Complex Fractions Revisited

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R.6 Example 1 Using the Definition of a Negative Exponent (page 53)

Evaluate each expression.

(a) 10^{-3} (b) -5^{-1} (c) $\left(\frac{4}{9}\right)^{-2}$

(a) $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$ (b) $-5^{-1} = -\frac{1}{5}$

(c) $\left(\frac{4}{9}\right)^{-2} = \frac{1}{\left(\frac{4}{9}\right)^2} = \frac{1}{\frac{16}{81}} = \frac{81}{16}$

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R.6 Example 1 Using the Definition of a Negative Exponent (cont.)

Write the expression without negative exponents.

(d) $mn^{-4} = \frac{m}{n^4}$

(e) $(mn)^{-4} = \frac{1}{(mn)^4} = \frac{1}{m^4 n^4}$

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R.6 Example 2 Using the Quotient Rule (page 54)

Simplify each expression.

(a) $\frac{15^8}{15^3} = 15^{8-3} = 15^5$ (b) $\frac{y^4}{y^{-9}} = y^{4-(-9)} = y^{13}$

(c) $\frac{35r^6}{25r^{-4}} = \frac{7r^{6-(-4)}}{5} = \frac{7r^{10}}{5}$

(d) $\frac{34a^8b^{11}}{51a^{12}b^5} = \frac{34}{51} \cdot \frac{a^8}{a^{12}} \cdot \frac{b^{11}}{b^5} = \frac{2}{3} a^{8-12} b^{11-5}$
 $= \frac{2}{3} a^{-4} b^6 = \frac{2b^6}{3a^4}$

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R.6 Example 3(a) Using Rules for Exponents (page 54)

Simplify.

$$\begin{aligned} 5x^3(2^{-1}x^4)^{-3} &= 5x^3(2^{-1(-3)}x^{4(-3)}) \\ &= 5x^3(2^3x^{-12}) \\ &= 5x^{3-12}(8) \\ &= 5x^{-9}(8) \\ &= \frac{40}{x^9} \end{aligned}$$

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R.6 Example 3(b) Using Rules for Exponents (page 54)

Simplify.

$$\begin{aligned} \frac{30r^4s^{-9}}{45r^{-6}s^3} &= \frac{30}{45} \cdot \frac{r^4}{r^{-6}} \cdot \frac{s^{-9}}{s^3} \\ &= \frac{2}{3} \cdot r^{4-(-6)} \cdot s^{-9-3} \\ &= \frac{2}{3} \cdot r^{10} \cdot s^{-12} \\ &= \frac{2r^{10}}{3s^{12}} \end{aligned}$$

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R.6 Example 3(c) Using Rules for Exponents (page 54)

Simplify.

$$\begin{aligned} \frac{(4b^3)^{-2}(4b^{-1})^{-3}}{(4^{-1}b^3)^{-4}} &= \frac{4^{-2}b^{3(-2)}(4^{-3})b^{-1(-3)}}{4^{-1(-4)}b^{3(-4)}} \\ &= \frac{4^{-2} \cdot 4^{-3} b^{-6} b^3}{4^4 b^{-12}} \\ &= \frac{4^{-2-3} b^{-6+3}}{4^4 b^{-12}} = \frac{4^{-5} b^{-3}}{4^4 b^{-12}} \\ &= 4^{-5-4} b^{-3-(-12)} = 4^{-9} b^9 = \frac{b^9}{4^9} \end{aligned}$$

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R.6 Example 4 Using the Definition of $a^{1/n}$ (page 55)

Evaluate each expression.

- (a) $49^{1/2} = 7$ (b) $-144^{1/2} = -12$
 (c) $-(144)^{1/2} = -12$ (d) $64^{1/6} = 2$
 (e) $(-64)^{1/6}$ (f) $-64^{1/6} = -2$
 not a real number
 (g) $(-125)^{1/3} = -5$ (h) $-64^{1/3} = -4$

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R.6 Example 5 Using the Definition of $a^{m/n}$ (page 56)

Evaluate each expression.

- (a) $81^{3/4} = (81^{1/4})^3 = 3^3 = 27$
 (b) $25^{3/2} = (25^{1/2})^3 = 5^3 = 125$
 (c) $-4^{5/2} = -(4^{1/2})^5 = -2^5 = -32$

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R.6 Example 5 Using the Definition of $a^{m/n}$ (cont.)

Evaluate each expression.

- (d) $(-64)^{2/3} = [(-64)^{1/3}]^2 = (-4)^2 = 16$
 (e) $216^{-2/3} = (216^{1/3})^{-2} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$
 (f) $(-100)^{3/2}$ is not a real number because $(-100)^{1/2}$ is not a real number.

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R.6 Example 6 Combining the Definitions and Rules for Exponents (page 57)

Simplify each expression.

$$(a) \frac{18^{1/2} \cdot 18^{7/2}}{18^3} = \frac{18^{1/2+7/2}}{18^3} = 18^{4-3} = 18^1 = 18$$

$$(b) 100^{3/2} \cdot 16^{-3/4} = (100^{1/2})^3 (16^{1/4})^{-3} = 10^3 \cdot 2^{-3} \\ = 10^3 \cdot \frac{1}{2^3} = \frac{10^3}{2^3} = \frac{1000}{8} = 125$$

$$(c) 4z^{3/4} \cdot 5z^{2/5} = 20z^{3/4+2/5} = 20z^{23/20}$$

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R.6 Example 6 Combining the Definitions and Rules for Exponents (cont.)

Simplify each expression.

$$(d) \left(\frac{5m^{4/3}}{n^{2/3}}\right)^2 \left(\frac{m^4}{8n^5}\right)^{1/3} = \frac{5^2 m^{8/3}}{n^{4/3}} \cdot \frac{m^{4/3}}{8^{1/3} n^{5/3}} \\ = \frac{25m^{8/3+4/3}}{2n^{4/3+5/3}} = \frac{25m^4}{2n^3}$$

$$(e) y^{3/7} (y^{4/7} - 5y^{11/7}) = y^{3/7+4/7} - 5y^{3/7+11/7} \\ = y - 5y^2$$

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R.6 Example 7 Factoring Expressions with Negative or Rational Exponents (page 58)

Factor out the least power of the variable or variable expression.

$$(a) 28y^{-5} + 21y^{-2} = 7y^{-5}(4 + 3y^3)$$

$$(b) 18n^{4/3} - 12n^{1/3} = 6n^{1/3}(3n - 2)$$

$$(c) (x+3)^{-2/5} - (x+3)^{3/5} = (x+3)^{-2/5} [1 - (x+3)] \\ = (x+3)^{-2/5} (-2-x)$$

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R.6 Example 8 Simplifying a Fraction with Negative Exponents (page 58)

Simplify. Write the result with only positive exponents.

$$\frac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}} = \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}} = \frac{\frac{x+y}{xy}}{\frac{y^2-x^2}{x^2y^2}} \\ = \frac{x^2y^2 \cdot \frac{x+y}{xy}}{x^2y^2 \cdot \frac{y^2-x^2}{x^2y^2}} = \frac{xy(x+y)}{y^2-x^2} \\ = \frac{xy(x+y)}{(y-x)(y+x)} = \frac{xy}{y-x} \quad \text{Divide out the common factor}$$

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R.7 Radical Expressions

Radical Notations • Simplified Radicals • Operations with Radicals • Rationalizing Denominators

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R.7 Example 1 Evaluating Roots (page 63)

Write each root using exponents and evaluate.

$$(a) \sqrt[3]{27} = 27^{1/3} = 3$$

$$(b) -\sqrt[4]{10,000} = -10,000^{1/4} = -10$$

$$(c) \sqrt[3]{-216} = (-216)^{1/3} = -6$$

$$(d) \sqrt[4]{-81} \text{ is not a real number.}$$

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R.7 Example 1 Evaluating Roots (cont.)

Write each root using exponents and evaluate.

$$(e) \sqrt[3]{\frac{125}{512}} = \left(\frac{125}{512}\right)^{1/3} = \frac{125^{1/3}}{512^{1/3}} = \frac{5}{8}$$

$$(f) -\sqrt[5]{-243} = -(-243)^{1/5} = -(-3) = 3$$

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R.7 Example 2 Converting From Rational Exponents to Radicals (page 63)

Write in radical form and simplify.

$$(a) 16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$$

$$(b) (-64)^{2/3} = (\sqrt[3]{-64})^2 = (-4)^2 = 16$$

$$(c) -121^{3/2} = -(\sqrt{121})^3 = -11^3 = -1331$$

$$(d) y^{7/8} = \sqrt[8]{y^7}, y \geq 0$$

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R.7 Example 2 Converting From Rational Exponents to Radicals (cont.)

Write in radical form and simplify.

$$(e) 7z^{4/5} = 7\sqrt[5]{z^4}$$

$$(f) 12q^{-1/4} = \frac{12}{\sqrt[4]{q}}, q > 0$$

$$(g) (5x + 2y)^{1/6} = \sqrt[6]{5x + 2y}$$

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R.7 Example 3 Converting From Radicals to Rational Exponents (page 63)

Write in exponential form.

$$(a) \sqrt[7]{n^3} = n^{3/7} \quad (b) \sqrt[4]{10x} = (10x)^{1/4}$$

$$(c) 15(\sqrt[3]{r})^4 = 15r^{4/3}$$

$$(d) -2\sqrt[5]{(3x^2)^8} = -2\sqrt[5]{3^8 x^{16}} = -2 \cdot 3^{8/5} x^{16/5}$$

$$(e) \sqrt[3]{r^2 + s^4} = (r^2 + s^4)^{1/3}$$

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R.7 Example 4 Using Absolute Value to Simplify Roots (page 64)

Simplify each expression.

$$(a) \sqrt{z^6} = \sqrt{(z^3)^2} = |z^3|$$

$$(b) \sqrt[7]{t^7} = t^{7/7} = t$$

$$(c) \sqrt{81r^8s^{10}} = |9r^4s^5| = 9r^4|s^5|$$

$$(d) \sqrt[4]{(-3)^4} = |-3| = 3$$

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R.7 Example 4 Using Absolute Value to Simplify Roots (cont.)

Simplify each expression.

$$(e) \sqrt[5]{m^{10}} = m^{10/5} = m^2$$

$$(f) \sqrt{(3x - 4)^2} = |3x - 4|$$

$$(g) \sqrt{x^2 - 10x + 25} = \sqrt{(x - 5)^2} = |x - 5|$$

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R.7 Example 5 Using the Rules for Radicals to Simplify Radical Expressions (page 65)

Simplify each expression.

$$(a) \sqrt{5} \cdot \sqrt{45} = \sqrt{5 \cdot 45} = \sqrt{225} = 15$$

$$(b) \sqrt[5]{n^3} \cdot \sqrt[5]{n^2} = \sqrt[5]{n^3 \cdot n^2} = \sqrt[5]{n^5} = n$$

$$(c) \sqrt{\frac{11}{169}} = \frac{\sqrt{11}}{\sqrt{169}} = \frac{\sqrt{11}}{13}$$

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R.7 Example 5 Using the Rules for Radicals to Simplify Radical Expressions (cont.)

Simplify each expression.

$$(d) \sqrt[6]{\frac{a}{b^{12}}} = \frac{\sqrt[6]{a}}{\sqrt[6]{b^{12}}} = \frac{\sqrt[6]{a}}{b^2}$$

$$(e) \sqrt[5]{4\sqrt{17}} = \sqrt[20]{17}$$

$$(f) \sqrt{\sqrt[6]{8}} = \sqrt[12]{8}$$

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R.7 Example 6 Simplifying Radicals (page 66)

Simplify each radical.

$$(a) \sqrt{288} = \sqrt{2 \cdot 144} = \sqrt{2} \cdot \sqrt{144} = 12\sqrt{2}$$

$$(b) -8\sqrt[3]{125} = -8 \cdot 5 = -40$$

$$(c) \sqrt[3]{128a^6b^8c^{10}} = \sqrt[3]{2 \cdot 64a^6b^6b^2c^9c} \\ = \sqrt[3]{(64a^6b^6c^9)(2b^2c)} \\ = 4a^2b^2c^3\sqrt[3]{2b^2c}$$

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R.7 Example 7 Adding and Subtracting Like Radicals (page 66)

Add or subtract as indicated.

$$(a) 14\sqrt{5pq} - 11\sqrt{5pq} = (14 - 11)\sqrt{5pq} \\ = 3\sqrt{5pq}$$

$$(b) \sqrt{75ab^3} - b\sqrt{12ab} = \sqrt{3 \cdot 25ab^2b} - b\sqrt{4 \cdot 3ab} \\ = 5b\sqrt{3ab} - 2b\sqrt{3ab} \\ = (5b - 2b)\sqrt{3ab} \\ = 3b\sqrt{3ab}$$

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R.7 Example 7 Adding and Subtracting Like Radicals (cont.)

Add or subtract as indicated.

$$(c) \sqrt[3]{81x^5y^7} + \sqrt[3]{24x^8y^4} \\ = \sqrt[3]{27 \cdot 3x^3x^2y^6y} + \sqrt[3]{8 \cdot 3x^6x^2y^3y} \\ = 3xy^2\sqrt[3]{3x^2y} + 2x^2y\sqrt[3]{3x^2y} \\ = (3xy^2 + 2x^2y)\sqrt[3]{3x^2y}$$

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R.7 Example 8 Simplifying Radicals by Writing Them With Rational Exponents (page 67)

Simplify each radical.

$$(a) \sqrt[10]{2^5} = 2^{5/10} = 2^{1/2} = \sqrt{2}$$

$$(b) \sqrt[3]{a^9b^{18}} = a^{9/3}b^{18/3} = a^3b^6$$

$$(c) \sqrt[6]{\sqrt[3]{4^2}} = \sqrt[18]{4^2} = 4^{2/18} = 4^{1/9} = \sqrt[9]{4}$$

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R.7 Example 9(a) Multiplying Radical Expressions (page 68)

Find the product.

$$(\sqrt{11} + \sqrt{17})(\sqrt{11} - \sqrt{17}) = (\sqrt{11})^2 - (\sqrt{17})^2 \quad \text{Product of the sum and difference of two terms.}$$

$$= 11 - 17 = -6$$

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R.7 Example 9(b) Multiplying Radical Expressions (page 68)

Find the product.

$$(5 + \sqrt{32})(3 - \sqrt{2}) = (5 + \sqrt{2 \cdot 16})(3 - \sqrt{2}) \quad \text{Simplify } \sqrt{32}$$

$$= (5 + 4\sqrt{2})(3 - \sqrt{2})$$

$$= 15 - 5\sqrt{2} + 12\sqrt{2} - 4\sqrt{2}\sqrt{2} \quad \text{FOIL}$$

$$= 15 - 5\sqrt{2} + 12\sqrt{2} - 8$$

$$= 7 + 7\sqrt{2}$$

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R.7 Example 10 Rationalizing Denominators (page 68)

Rationalize each denominator.

$$(a) \frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

$$(b) \sqrt[3]{\frac{4}{9}} = \frac{\sqrt[3]{4}}{\sqrt[3]{9}} = \frac{\sqrt[3]{4} \cdot \sqrt[3]{3}}{\sqrt[3]{3^2} \cdot \sqrt[3]{3}} = \frac{\sqrt[3]{12}}{\sqrt[3]{3^3}} = \frac{\sqrt[3]{12}}{3}$$

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R.7 Example 11(a) Simplifying Radical Expressions with Fractions (page 69)

Simplify the expression.

$$\frac{\sqrt[3]{a^5b}}{\sqrt[3]{a^2b^5}} = \sqrt[3]{\frac{a^5b}{a^2b^5}} = \sqrt[3]{\frac{a^3}{b^4}} = \frac{\sqrt[3]{a^3}}{\sqrt[3]{b^4}} \quad \text{Quotient rule}$$

$$= \frac{a}{b\sqrt[3]{b}} = \frac{a\sqrt[3]{b^2}}{b\sqrt[3]{b}\sqrt[3]{b^2}} \quad \text{Rationalize denominator.}$$

$$= \frac{a\sqrt[3]{b^2}}{b\sqrt[3]{b^3}} = \frac{a\sqrt[3]{b^2}}{b \cdot b} = \frac{a\sqrt[3]{b^2}}{b^2}$$

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R.7 Example 11(b) Simplifying Radical Expressions with Fractions (page 69)

Simplify the expression.

$$\sqrt[4]{\frac{6}{x^8}} - \sqrt[4]{\frac{3}{x^{16}}} = \frac{\sqrt[4]{6}}{\sqrt[4]{x^8}} - \frac{\sqrt[4]{3}}{\sqrt[4]{x^{16}}} \quad \text{Quotient rule}$$

$$= \frac{\sqrt[4]{6}}{x^2} - \frac{\sqrt[4]{3}}{x^4} \quad \text{Simplify the denominators.}$$

$$= \frac{x^2\sqrt[4]{6}}{x^4} - \frac{\sqrt[4]{3}}{x^4} \quad \text{Write with a common denominator.}$$

$$= \frac{x^2\sqrt[4]{6} - \sqrt[4]{3}}{x^4} \quad \text{Subtract the numerators.}$$

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R.7 Example 12 Rationalizing a Binomial Denominator (page 69)

Rationalize the denominator.

$$\frac{2}{3 + \sqrt{5}} = \frac{2(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})} \quad \text{Multiply the numerator and denominator by the conjugate of the denominator.}$$

$$= \frac{2(3 - \sqrt{5})}{3^2 - (\sqrt{5})^2} = \frac{2(3 - \sqrt{5})}{9 - 5}$$

$$= \frac{2(3 - \sqrt{5})}{4} = \frac{3 - \sqrt{5}}{2}$$

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