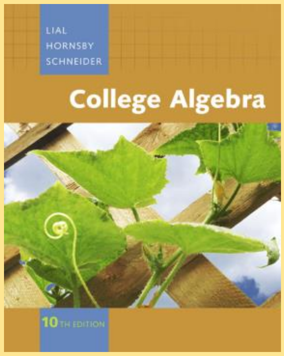
5

Systems and Matrices

Sections 5.6–5.8



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5 Systems and Matrices

5.6 Systems of Inequalities and Linear Programming

Solving Linear Inequalities ▪ Solving Systems of Inequalities ▪ Linear Programming

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5.6 Example 1 Graphing a Linear Inequality (page 555)

Graph $3x - 2y \geq 6$.

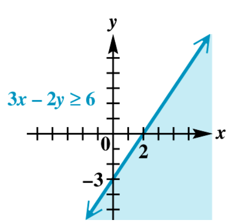
The boundary of the graph is the straight line $3x - 2y = 6$, which can be graphed using the *x*-intercept 2 and the *y*-intercept –3. The boundary is included in the graph, so draw a straight line.

Solve the equation for *y*.

$$3x - 2y \geq 6 \Rightarrow -2y \geq -3x + 6 \Rightarrow y \leq \frac{3}{2}x - 3$$

5.6 Example 1 Graphing a Linear Inequality (cont.)

The graph of the solution set is the half-plane below the boundary.



5.6 Example 1 Graphing a Linear Inequality (cont.)

Check

Choose a test point not on the boundary line and substitute its coordinates into the inequality.

Test point: (0, 0)

$$3(0) - 2(0) \geq 6 \Rightarrow 0 \geq 6 \quad \text{False}$$

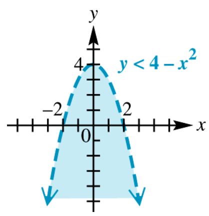
The point (0, 0) lies above the boundary and is not included in the solution set, which agrees with the graph.

5.6 **Example 2(a)** Graphing Systems of Inequalities (page 556)

Graph the system

$$y < 4 - x^2 \quad (1)$$
$$x < y - 1 \quad (2)$$

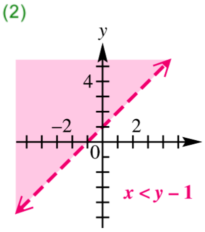
The graph of inequality (1) is a dashed parabola with vertex (0, 4) and x-intercepts (–2, 0) and (2, 0). Shade the region inside the parabola.



5.6 **Example 2(a)** Graphing Systems of Inequalities (cont.)

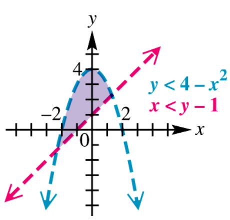
$$y < 4 - x^2 \quad (1)$$
$$x < y - 1 \quad (2)$$

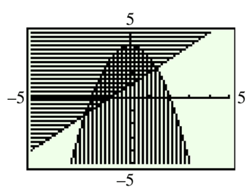
The graph of inequality (2) is a dashed line with x-intercept –1 and y-intercept 1. Shade the region above the line.



5.6 **Example 2(a)** Graphing Systems of Inequalities (cont.)

The graph of the solution set is the common region.



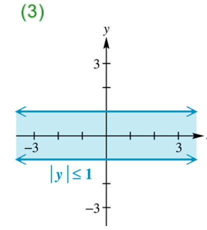


5.6 **Example 2(b)** Graphing Systems of Inequalities (page 556)

Graph the system

$$|y| \le 1 \quad (1)$$
$$x \ge 0 \quad (2)$$
$$y > 2|x| + 1 \quad (3)$$

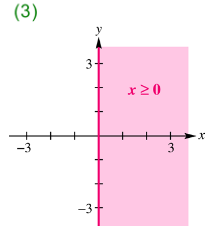
$|y| \le 1 \Rightarrow -1 \le y \le 1$, so the graph consists of the points between and on the lines $y = 1$ and $y = –1$.



5.6 **Example 2(b)** Graphing Systems of Inequalities (cont.)

$$|y| \le 1 \quad (1)$$
$$x \ge 0 \quad (2)$$
$$y > 2|x| + 1 \quad (3)$$

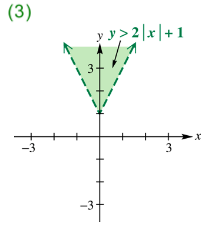
The graph of $x \ge 0$ includes the points on or to the right of the y-axis.



5.6 **Example 2(b)** Graphing Systems of Inequalities (cont.)

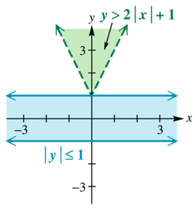
$$|y| \le 1 \quad (1)$$
$$x \ge 0 \quad (2)$$
$$y > 2|x| + 1 \quad (3)$$

The graph of $y > 2|x| + 1$ is the set of points inside the boundary $y = 2|x| + 1$.



5.6 **Example 2(b)** Graphing Systems of Inequalities (page 556)

Since the solution sets of $|y| \leq 1$ (1) and $y > 2|x| + 1$ (3) have no points in common, the solution set is ø.



5.6 **Example 3** Finding a Maximum Profit Model (page 557)

A company makes two products – MP3 players and DVD players. Each MP3 player gives a profit of \$50, and each DVD player gives a profit of \$20. The company must manufacture at least 40, but no more than 60, MP3 players per day. The number of DVD players cannot exceed 75 per day, and the number of MP3 players cannot exceed the number of DVD players. How many of each should the company manufacture to obtain the maximum profit?

Let x = the number of MP3 players produced daily
Let y = the number of DVD players produced daily

5.6 **Example 3** Finding a Maximum Profit Model (cont.)

The company must produce at least 40 MP3 players, so $x \geq 40$.

Since no more than 60 MP3 players can be produced, $x \leq 60$.

No more than 75 DVD players can be produced, so $y \leq 75$.

The number of MP3 players cannot exceed the number of DVD players, so $x \leq y$.

The number of MP3 players and the number of DVD players cannot be negative, so $x \geq 0$ and $y \geq 0$.

5.6 **Example 3** Finding a Maximum Profit Model (cont.)

The constraints form the system

$x \geq 40$ (1)
$x \leq 60$ (2)
$y \geq 75$ (3)
$x \leq y$ (4)
$x \geq 0$ (5)
$y \geq 0$ (6)

5.6 **Example 3** Finding a Maximum Profit Model (cont.)

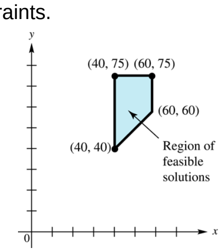
Each MP3 player give a profit of \$50, so the daily profit from production of x MP3 players is $50x$.

The profit from production of y DVD players is $20y$.

Thus, the total daily profit is $50x + 20y$. This is the function to be maximized, the **objective function**.

5.6 **Example 3** Finding a Maximum Profit Model (cont.)

To find the maximum possible profit, graph each constraint. The graph of the feasible region is the intersection of the regions that are the graphs of the individual constraints.



5.6 Example 3 Finding a Maximum Profit Model (cont.)

From the graph, we see that there are four vertices (40, 40), (40, 75), (60, 75), and (60, 60). Evaluate the objective function at each vertex to find the maximum possible value.

Point	Profit = 50x + 20y	
(40, 40)	50(40) + 20(40) = 2800	
(40, 75)	50(40) + 20(75) = 3500	
(60, 75)	50(60) + 20(75) = 4500	← Maximum
(60, 60)	50(60) + 20(60) = 4200	

The maximum profit of $4500 will be reached when 60 MP3 players and 75 DVD players are produced.

5.6 Example 4 Finding a Minimum Cost Model (page 559)

Robin takes vitamin pills each day. She wants at least 16 units of Vitamin A, at least 5 units of Vitamin B_1, and at least 20 units of Vitamin C daily. She can choose between red pills, costing 20¢ each, that contain 8 units of A, 1 of B_1, and 2 of C, or blue pills, costing 10¢ each, that contain 2 units of A, 1 of B_1, and 7 of C. How many of each pill should she take each day to minimize her cost and yet fulfill her daily requirements?

Let x = the number of red pills to buy
Let y = the number of blue pills to buy

5.6 Example 4 Finding a Minimum Cost Model (page 559)

Robin gets 8x units of vitamin A from the red pills and 2y units of vitamin A from the blue pills. Since she wants at least 16 units of vitamin A per day, $8x + 2y \geq 16$.

Each red pill and each blue pill supplies 1 unit of vitamin B_1. Robin wants at least 5 units per day, so $x + y \geq 5$.

Robin gets 2x units of vitamin C from the red pills and 7y units of vitamin C from the blue pills. Since she wants at least 20 units of vitamin C per day, $2x + 7y \geq 20$.

5.6 Example 4 Finding a Minimum Cost Model (cont.)

Robin cannot buy negative numbers of the pills, so $x \geq 0$ and $y \geq 0$.

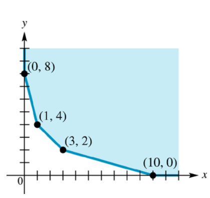
The constraints form the system

$$
\begin{aligned}
8x + 2y &\geq 16 \quad (1)\\
x + y &\geq 5 \quad (2)\\
2x + 7y &\geq 20 \quad (3)\\
x &\geq 0 \quad (4)\\
y &\geq 0 \quad (5)
\end{aligned}
$$

Thus, the total daily cost is $20x + 10y$. This is the function to be maximized, the **objective function**.

5.6 Example 3 Finding a Minimum Cost Model (cont.)

To find the minimum possible cost, graph each constraint. The graph of the feasible region is the intersection of the regions that are the graphs of the individual constraints.



5.6 Example 4 Finding a Minimum Cost Model (cont.)

From the graph, we see that the vertices are (0, 8), (1, 4), (3, 2), and (10, 0). Evaluate the objective function at each vertex.

Point	Cost = 20x + 10y	
(0, 8)	20(0) + 10(8) = 80	
(1, 4)	20(1) + 10(4) = 60	← Minimum
(3, 2)	20(3) + 10(2) = 80	
(10, 0)	20(10) + 10(0) = 200	

The minimum cost of 60¢ will be obtained when she takes 1 red pill and 4 blue pills per day.

5.7 Properties of Matrices

Basic Definitions ▪ Adding Matrices ▪ Special Matrices ▪ Subtracting Matrices ▪ Multiplying Matrices ▪ Applying Matrix Algebra

5.7 Example 1 Finding Values to Make Two Matrices Equal (page 565)

Find the values of the variables for which each statement is true, if possible.

(a) $\begin{bmatrix} a & b \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 9 \\ c & d \end{bmatrix}$

Since corresponding elements are equal, $a = -3$, $b = 9$, $c = -5$, and $d = 0$.

(b) $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

The statement cannot be true since $\begin{bmatrix} x & y \end{bmatrix}$ is a 1 × 2 matrix, while $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$ is a 2 × 1 matrix.

5.7 Example 2 Adding Matrices (page 566)

Find each sum, if possible.

(a) $\begin{bmatrix} 3 & -8 \\ -4 & 6 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ 10 & -6 \end{bmatrix} = \begin{bmatrix} 3+7 & -8+(-5) \\ -4+10 & 6+(-6) \end{bmatrix}$

$= \begin{bmatrix} 10 & -13 \\ 6 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} -9 \\ 7 \\ 6 \\ -3 \end{bmatrix} + \begin{bmatrix} -9 \\ 5 \\ -6 \\ 10 \end{bmatrix} = \begin{bmatrix} -18 \\ 12 \\ 0 \\ 7 \end{bmatrix}$

5.7 Example 2 Adding Matrices (cont.)

(c) $A + B$ if $A = \begin{bmatrix} 3 & -1 & 6 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 8 \\ 3 & -1 \\ -5 & 2 \end{bmatrix}$

A and *B* cannot be added because *A* is a 2 × 3 matrix, while *B* is a 3 × 2 matrix.

5.7 Example 3 Subtracting Matrices (page 567)

Find each difference, if possible.

(a) $\begin{bmatrix} 8 & -9 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} -8 & 4 \\ -7 & 11 \end{bmatrix} = \begin{bmatrix} 8-(-8) & -9-4 \\ -6-(-7) & 2-11 \end{bmatrix}$

$= \begin{bmatrix} 16 & -13 \\ 1 & -9 \end{bmatrix}$

(b) $\begin{bmatrix} 9 \\ -3 \\ 6 \end{bmatrix} - \begin{bmatrix} 18 \\ 12 \\ -6 \end{bmatrix} = \begin{bmatrix} -9 \\ -15 \\ 12 \end{bmatrix}$

5.7 Example 3 Subtracting Matrices (cont.)

(c) $A - B$ if $A = \begin{bmatrix} 4 & 5 & 0 \\ -2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ 3 & -2 \end{bmatrix}$

A and *B* cannot be subtracted because *A* is a 2 × 3 matrix, while *B* is a 2 × 2 matrix.

5.7 Example 4 Multiplying Matrices by Scalars (page 568)

Find each product.

(a) $-3\begin{bmatrix} 2 & -5 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} -3(2) & -3(-5) \\ -3(-4) & -3(0) \end{bmatrix} = \begin{bmatrix} -6 & 15 \\ 12 & 0 \end{bmatrix}$

(b) $\frac{4}{5}\begin{bmatrix} -25 & 10 \\ 15 & -45 \end{bmatrix} = \begin{bmatrix} -20 & 8 \\ 12 & -36 \end{bmatrix}$

5.7 Example 5 Deciding Whether Two Matrices Can Be Multiplied (page 570)

Suppose C is a 2 × 5 matrix and D is a 4 × 2 matrix.

(a, b) Can the product CD be calculated? If so, what size is it?

No, CD cannot be calculated.

Matrix C Matrix D

2×5 4×2

different

(c, d) Can the product DC be calculated? If so, what size is it?

Yes, DC can be calculated. The result is a 4 × 5 matrix.

Matrix D Matrix C

4×2 2×5

matches

size of DC 4×5

5.7 Example 6 Multiplying Matrices (page 571)

Let $A = \begin{bmatrix} 3 & -4 & 5 \\ 1 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ -3 & 8 \end{bmatrix}$.

Find each product, if possible.

(a) AB AB cannot be calculated.

Matrix A Matrix B

2×3 2×2

different

(b) BA

A 2 × 2 matrix multiplied by a 2 × 3 matrix results in a 2 × 3 matrix.

5.7 Example 6 Multiplying Matrices (cont.)

$$\begin{bmatrix} 4 & 1 \\ -3 & 8 \end{bmatrix}\begin{bmatrix} 3 & -4 & 5 \\ 1 & 0 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 4(3)+1(1) & 4(-4)+1(0) & 4(5)+1(-6) \\ -3(3)+8(1) & -3(-4)+8(0) & -3(5)+8(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -16 & 14 \\ -1 & 12 & -63 \end{bmatrix}$$

5.7 Example 7 Multiplying Square Matrices in Different Orders (page 571)

Let $C = \begin{bmatrix} 2 & -5 \\ -6 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & -4 \\ 2 & 3 \end{bmatrix}$. Find each product.

(a) CD

$$\begin{bmatrix} 2 & -5 \\ -6 & 1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2(3)+(-5)(2) & 2(-4)+(-5)(3) \\ -6(3)+1(2) & -6(-4)+1(3) \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -23 \\ -16 & 27 \end{bmatrix}$$

5.7 Example 7 Multiplying Square Matrices in Different Orders (cont.)

(b) DC

$$\begin{bmatrix} 3 & -4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ -6 & 1 \end{bmatrix} = \begin{bmatrix} 3(2)+(-4)(-6) & 3(-5)+(-4)(1) \\ 2(2)+3(-6) & 2(-5)+3(1) \end{bmatrix}$$

$$= \begin{bmatrix} 30 & -19 \\ -14 & -7 \end{bmatrix}$$

5.7 Example 8 Using Matrix Multiplication to Model Plans for a Subdivision (page 572)

A contractor builds three kinds of houses, models A, B, and C, with a choice of two styles, colonial or ranch. Matrix *M* shows the number of each kind of house the contractor is planning to build for a new 150-home subdivision. The amounts for each of the main materials used depend on the style of the house. These amounts are shown in matrix *Q*, while matrix *R* gives the cost in dollars for each kind of material. Concrete is measured here in cubic yards, lumber in 1000 board feet, brick in 1000s, and shingles in 100 square feet.

5.7 Example 8 Using Matrix Multiplication to Model Plans for a Subdivision (cont.)

$$\begin{array}{c} \\ \text{Model A} \\ \text{Model B} \\ \text{Model C} \end{array} \begin{array}{c} \begin{array}{cc} \text{Colonial} & \text{Ranch} \end{array} \\ \begin{bmatrix} 10 & 30 \\ 15 & 25 \\ 25 & 45 \end{bmatrix} \end{array} = M$$

$$\begin{array}{c} \text{Concrete} \\ \text{Lumber} \\ \text{Brick} \\ \text{Shingles} \end{array} \overset{\text{Cost per unit}}{\begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix}} = R$$

$$\begin{array}{c} \\ \text{Colonial} \\ \text{Ranch} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Concrete} & \text{Lumber} & \text{Brick} & \text{Shingles} \end{array} \\ \begin{bmatrix} 10 & 2 & 0 & 2 \\ 50 & 1 & 20 & 2 \end{bmatrix} \end{array} = Q$$

5.7 Example 8 Using Matrix Multiplication to Model Plans for a Subdivision (cont.)

(a) What is the total cost of materials for all houses of each model?

To calculate the total cost of material for all houses of each model, first find *MQ*, which will show the total amount of each material needed for all houses of each model.

5.7 Example 8 Using Matrix Multiplication to Model Plans for a Subdivision (cont.)

$$MQ = \begin{bmatrix} 10 & 30 \\ 15 & 25 \\ 25 & 45 \end{bmatrix} \begin{bmatrix} 10 & 2 & 0 & 2 \\ 50 & 1 & 20 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10(10)+30(50) & 10(2)+30(1) & 10(0)+30(20) & 10(2)+30(2) \\ 15(10)+25(50) & 15(2)+25(1) & 15(0)+25(20) & 15(2)+25(2) \\ 25(10)+45(50) & 25(2)+45(1) & 25(0)+45(20) & 25(2)+45(2) \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{cccc} \text{Concrete} & \text{Lumber} & \text{Brick} & \text{Shingles} \end{array} \\ \begin{bmatrix} 1600 & 50 & 600 & 80 \\ 1400 & 55 & 500 & 80 \\ 2500 & 95 & 900 & 140 \end{bmatrix} \end{array}$$

5.7 Example 8 Using Matrix Multiplication to Model Plans for a Subdivision (cont.)

Multiplying *MQ* and the cost matrix *R* gives the total cost of material for each model.

$$(MQ)R = \begin{bmatrix} 1600 & 50 & 600 & 80 \\ 1400 & 55 & 500 & 80 \\ 2500 & 95 & 900 & 140 \end{bmatrix} \begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix}$$

$$= \begin{bmatrix} 1600(20)+50(180)+600(60)+80(25) \\ 1400(20)+55(180)+500(60)+80(25) \\ 2500(20)+95(180)+900(60)+140(25) \end{bmatrix} = \begin{bmatrix} 79{,}000 \\ 69{,}900 \\ 124{,}600 \end{bmatrix}$$

The cost of materials for model A is $79,000. For model B, the cost is $69,900. For model C, the cost is $124,600.

5.7 Example 8 Using Matrix Multiplication to Model Plans for a Subdivision (cont.)

(b) How much of each of the four kinds of material must be ordered?

To find how much of each kind of material to order, total each column of matrix *MQ*. Write this as a row matrix,

$$MQ = \begin{array}{c} \begin{array}{cccc} \text{Concrete} & \text{Lumber} & \text{Brick} & \text{Shingles} \end{array} \\ \begin{bmatrix} 1600 & 50 & 600 & 80 \\ 1400 & 55 & 500 & 80 \\ 2500 & 95 & 900 & 140 \end{bmatrix} \end{array}$$

$$T = \begin{bmatrix} 5500 & 200 & 2000 & 300 \end{bmatrix}$$

5500 units of concrete, 200 units of lumber, 2000 units of brick and 300 units of shingles must be ordered.

5.7 Example 8 Using Matrix Multiplication to Model Plans for a Subdivision (cont.)

(c) What is the total cost of the materials?

To find the total cost of the materials, find *TR.*

$$TR = \begin{bmatrix} 20 \\ 180 \\ 60 \\ 25 \end{bmatrix} \begin{bmatrix} 5500 & 200 & 2000 & 300 \end{bmatrix}$$

$$= \begin{bmatrix} 20(5500) + 180(200) + 60(2000) + 25(300) \end{bmatrix}$$

$$= \begin{bmatrix} 273{,}500 \end{bmatrix}$$

The total cost of the materials is $273,500.

5.8 Matrix Inverses

Identity Matrices ▪ Multiplicative Inverses ▪ Solving Systems Using Inverse Matrices

5.8 Example 1 Verifying the Identity Property of I_3 (page 580)

Let $B = \begin{bmatrix} 3 & -5 & 8 \\ 2 & 0 & 6 \\ -4 & 1 & -7 \end{bmatrix}$. Show that $I_3B = B$.

The 3 ×3 identity matrix is

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5.8 Example 1 Verifying the Identity Property of I_3 (cont.)

$$I_3B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 & 8 \\ 2 & 0 & 6 \\ -4 & 1 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 1(3)+0(2)+0(-4) & 1(-5)+0(0)+0(1) & 1(8)+0(6)+0(-7) \\ 0(3)+1(2)+0(-4) & 0(-5)+1(0)+0(1) & 0(8)+1(6)+0(-7) \\ 0(3)+0(2)+1(-4) & 0(-5)+0(0)+1(1) & 0(8)+0(6)+1(-7) \end{bmatrix}$$

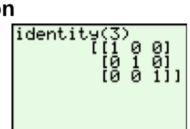
$$= \begin{bmatrix} 3+0+0 & -5+0+0 & 8+0+0 \\ 0+2+0 & 0+0+0 & 0+6+0 \\ 0+0+(-4) & 0+0+1 & 0+0+(-7) \end{bmatrix}$$

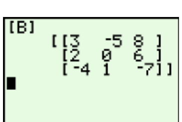
$$= \begin{bmatrix} 3 & -5 & 8 \\ 2 & 0 & 6 \\ -4 & 1 & -7 \end{bmatrix} = B$$

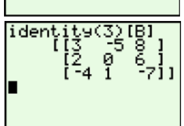
5.8 Example 1 Verifying the Identity Property of I_3 (cont.)

Graphing calculator solution

The identity matrix for $n = 3$.







The graphing calculator screens support the algebraic solution.

5.8 Example 2 Finding the Inverse of a 3 × 3 Matrix (page 583)

Find B^{-1} if $B = \begin{bmatrix} -4 & 2 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$.

Write the augmented matrix $[B|I_3]$:

$$\left[\begin{array}{ccc|ccc} -4 & 2 & 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] -\frac{1}{4}R1$$

1 in first row, first column

5.8 Example 2 Finding the Inverse of a 3 × 3 Matrix (page 583)

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & -2 & -\frac{1}{4} & -1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ R1-R2 \\ \\ \end{matrix}$$

0 in second row, first column

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & -2 & -\frac{1}{4} & -1 & 0 \\ 0 & 0 & -8 & -\frac{1}{2} & -2 & -1 \end{array}\right] \begin{matrix} \\ \\ 2R2-R3 \end{matrix}$$

0 in third row, second column

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 1 & -4 & -\frac{1}{2} & -2 & 0 \\ 0 & 0 & 1 & \frac{1}{16} & \frac{1}{4} & \frac{1}{8} \end{array}\right] \begin{matrix} \\ 2R2 \\ -\frac{1}{8}R3 \end{matrix}$$

1 in second row, second column; 1 in third row, third column

5.8 Example 2 Finding the Inverse of a 3 × 3 Matrix (page 583)

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{4} & -1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{16} & \frac{1}{4} & \frac{1}{8} \end{array}\right] \begin{matrix} \\ R2+4R3 \\ \\ \end{matrix}$$

0 in second row, third column

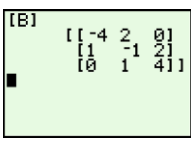
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{8} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{4} & -1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{16} & \frac{1}{4} & \frac{1}{8} \end{array}\right] \begin{matrix} R1+\frac{1}{2}R2 \\ \\ \\ \end{matrix}$$

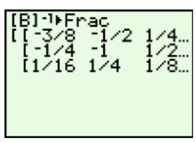
0 in first row, second column

$$B^{-1} = \begin{bmatrix} -\frac{3}{8} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{4} & -1 & \frac{1}{2} \\ \frac{1}{16} & \frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

5.8 Example 2 Finding the Inverse of a 3 × 3 Matrix (page 583)

Graphing calculator solution





The graphing calculator screens support the algebraic solution.

5.8 Example 3 Identifying a Matrix With No Inverse (page 584)

Find A^{-1} given that $A = \begin{bmatrix} 4 & -2 & 5 \\ 0 & 1 & 0 \\ -8 & 4 & -10 \end{bmatrix}$.

Write the augmented matrix $[A \mid I_3]$:

$$\left[\begin{array}{ccc|ccc} 4 & -2 & 5 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -8 & 4 & -10 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 4 & -2 & 5 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ 2R1+R3 \end{matrix}$$

Since there is no way to convert the third element in the third row to a 1, A^{-1} does not exist.

5.8 Example 4(a) Solving Systems of Equations Using Matrix Inverses (page 585)

Use the inverse of the coefficient matrix to solve the system.

$$5x + 2y = -1$$
$$2x + 3y = 15$$

Write the system in matrix form.

$$\begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 15 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ 15 \end{bmatrix}$$

5.8 Example 4(a) Solving Systems of Equations Using Matrix Inverses (cont.)

Find A^{-1}:

$$[A \mid I_2] = \left[\begin{array}{cc|cc} 5 & 2 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right]$$

Write the augmented matrix.

$$\left[\begin{array}{cc|cc} 1 & \frac{2}{5} & \frac{1}{5} & 0 \\ 2 & 3 & 0 & 1 \end{array}\right] \begin{matrix} \frac{1}{5}R1 \\ \\ \end{matrix}$$

1 in first row, first column

$$\left[\begin{array}{cc|cc} 1 & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{11}{5} & -\frac{2}{5} & 1 \end{array}\right] \begin{matrix} \\ -2R1+R2 \end{matrix}$$

0 in second row, first column

$$\left[\begin{array}{cc|cc} 1 & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -\frac{2}{11} & \frac{5}{11} \end{array}\right] \begin{matrix} \\ \frac{5}{11}R2 \end{matrix}$$

1 in second row, second column

5.8 Example 4(a) Solving Systems of Equations Using Matrix Inverses (cont.)

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{11} & -\frac{2}{11} \\ 0 & 1 & -\frac{2}{11} & \frac{5}{11} \end{array}\right] \text{R1} - \tfrac{2}{5}\text{R2}$$

0 in first row, second column

$$A^{-1} = \begin{bmatrix} \frac{3}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{5}{11} \end{bmatrix}$$

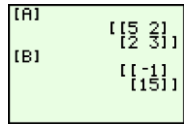
Now find $X = A^{-1}B$:

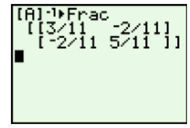
$$X = \begin{bmatrix} \frac{3}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{5}{11} \end{bmatrix}\begin{bmatrix} -1 \\ 15 \end{bmatrix} = \begin{bmatrix} \frac{3}{11}(-1) + \left(-\frac{2}{11}\right)(15) \\ -\frac{2}{11}(-1) + \frac{5}{11}(15) \end{bmatrix} = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$

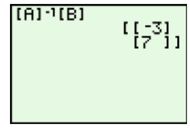
Solution set: {(–3, 7)}

5.8 Example 4(a) Solving Systems of Equations Using Matrix Inverses (cont.)

Graphing calculator solution







The graphing calculator screens support the algebraic solution.

5.8 Example 4(b) Solving Systems of Equations Using Matrix Inverses (page 585)

Use the inverse of the coefficient matrix to solve the system.

$$\begin{aligned} -4x + 2y \quad &= 12 \\ x - \ y + 2z &= 7 \\ y + 4z &= 20 \end{aligned}$$

Write the system in matrix form.

$$\begin{bmatrix} -4 & 2 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \\ 20 \end{bmatrix}$$

$$A = \begin{bmatrix} -4 & 2 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 12 \\ 7 \\ 20 \end{bmatrix}$$

5.8 Example 4(b) Solving Systems of Equations Using Matrix Inverses (cont.)

Find A^{-1}:

$$[A|I_3] = \left[\begin{array}{ccc|ccc} -4 & 2 & 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right]$$

Write the augmented matrix.

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] -\tfrac{1}{4}\text{R1}$$

1 in first row, first column

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & -2 & -\frac{1}{4} & -1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \text{R1} - \text{R2}$$

0 in second row, first column

5.8 Example 4(b) Solving Systems of Equations Using Matrix Inverses (cont.)

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 2 & 0 & -\frac{1}{2} & -2 & 1 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] 2\text{R2} + \text{R3}$$

0 in second row, third column

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{4} & -1 & \frac{1}{2} \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \tfrac{1}{2}\text{R2}$$

1 in second row, second column

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{8} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{4} & -1 & \frac{1}{2} \\ 0 & 0 & 4 & \frac{1}{4} & 1 & \frac{1}{2} \end{array}\right] \begin{array}{l} \text{R1} + \frac{1}{2}\text{R2} \\ \\ \text{R3} - \text{R2} \end{array}$$

0 in first row, second column;
0 in third row, second column

5.8 Example 4(b) Solving Systems of Equations Using Matrix Inverses (cont.)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{8} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{4} & -1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{16} & \frac{1}{4} & \frac{1}{8} \end{array}\right] \tfrac{1}{4}\text{R3}$$

1 in third row, third column

$$A^{-1} = \begin{bmatrix} -\frac{3}{8} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{4} & -1 & \frac{1}{2} \\ \frac{1}{16} & \frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

5.8 Example 4(b) Solving Systems of Equations Using Matrix Inverses (cont.)

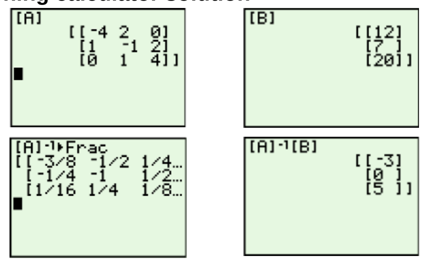
Now find $X = A^{-1}B$:

$$X = \begin{bmatrix} -\frac{3}{8} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{4} & -1 & \frac{1}{2} \\ \frac{1}{16} & \frac{1}{4} & \frac{1}{8} \end{bmatrix} \begin{bmatrix} 12 \\ 7 \\ 20 \end{bmatrix} = \begin{bmatrix} -\frac{3}{8}(12) - \frac{1}{2}(7) + \frac{1}{4}(20) \\ -\frac{1}{4}(12) - 1(7) + \frac{1}{2}(20) \\ \frac{1}{16}(12) + \frac{1}{4}(7) + \frac{1}{8}(20) \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 5 \end{bmatrix}$$

Solution set: {(–3, 0, 5)}

5.8 Example 4(b) Solving Systems of Equations Using Matrix Inverses (cont.)

Graphing calculator solution



The graphing calculator screens support the algebraic solution.