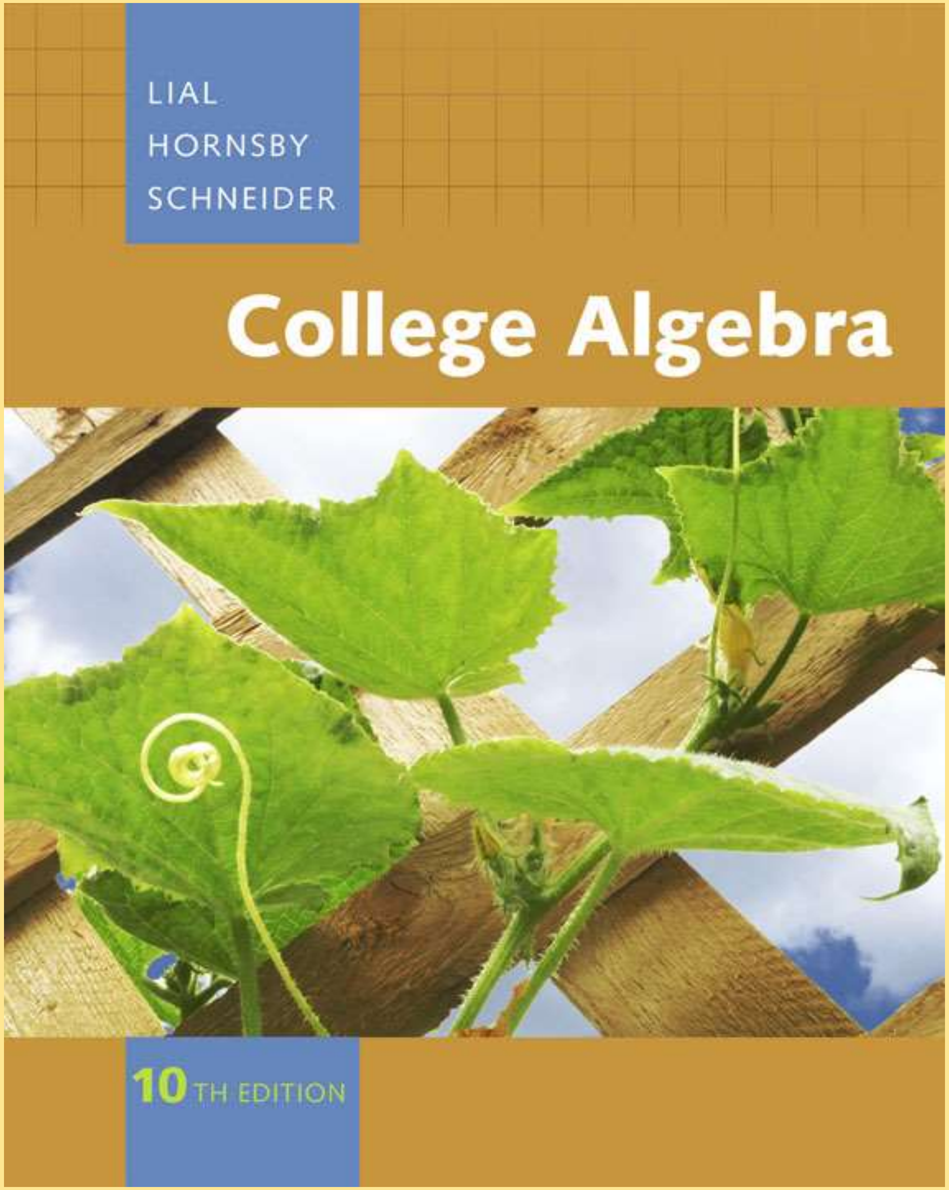
R

Review of Basic Concepts

Sections R.1–R.4



R Review of Basic Concepts

R.1 Sets

R.2 Real Numbers and Their Properties

R.3 Polynomials

R.4 Factoring Polynomials

R.1 Sets

Basic Definitions ▪ Operations on Sets

R.1 **Example 1** Listing the Elements of a Set (page 3)

Write the elements belonging to:

(a) $\{x|x$ is a natural number between 8 and 12$\}$

{9, 10, 11}

(b) $\{x|x$ is a state that borders Iowa$\}$

{Nebraska, South Dakota, Minnesota, Wisconsin, Missouri}

R.1 **Example 2** Finding the Complement of a Set (page 4)

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$A = \{2, 4, 6, 8\}$, $B = \{3, 6, 9\}$

Find A', B', U', $\varnothing'$

A' contains the elements of U that are not in A:
$\{1, 3, 5, 7, 9\}$

B' contains the elements of U that are not in B:
$\{1, 2, 4, 5, 7, 8\}$

$U' = \varnothing$ $\qquad$ $\varnothing' = U$

R.1 **Example 3** Finding the Intersection of Two Sets (page 4)

Find

(a) $\{15, 20, 25, 30\} \cap \{12, 18, 24, 30\}$

$\{15, 20, 25, 30\} \cap \{12, 18, 24, 30\} = \{30\}$

The element 30 is the only one belonging to both sets.

R.1 Example 3 Finding the Intersection of Two Sets (cont.)

Find

(b) $\{3, 6, 9, 12, 15, 18\} \cap \{6, 12, 18, 24\}$

$\{3, 6, 9, 12, 15, 18\} \cap \{6, 12, 18, 24\} = \{6, 12, 18\}$

The elements 6, 12, and 18 belong to both sets.

R.1 **Example 4** Finding the Union of Two Sets (page 4)

Find

(a) {1, 3, 5, 7, 9} U {3, 6, 9, 12}

List the elements of the first set, then include the elements from the second set that are not already listed.

{1, 3, 5, 7, 9} U {3, 6, 9, 12} = {1, 3, 5, 6, 7, 9, 12}

R.1 Example 4 Finding the Union of Two Sets (cont.)

Find

(b) {9, 10, 11, 12} U {10, 12, 14, 16}

{9, 10, 11, 12} U {10, 12, 14, 16}
= {9, 10, 11, 12, 14, 16}

R.2 Real Numbers and Their Properties

Sets of Numbers and the Number Line ▪ Exponents ▪ Order of Operations ▪ Properties of Real Numbers ▪ Order on the Number Line ▪ Absolute Value

R.2 Example 1 Identifying Elements of Subsets of the Real Numbers (page 8)

$$\text{Let } S = \left\{-12, -7.25, -\sqrt{3}, -\frac{5}{6}, 0, .\overline{45}, \sqrt{9}, \frac{21}{3}, 999\right\}$$

List the elements of *S* that belong to each set.

(a) natural numbers $\left\{\sqrt{9} \text{ (or 3)}, \frac{21}{3} \text{ (or 7)}, 999\right\}$

(b) whole numbers $\left\{0, \sqrt{9} \text{ (or 3)}, \frac{21}{3} \text{ (or 7)}, 999\right\}$

R.2 Example 1 Identifying Elements of Subsets of the Real Numbers (cont.)

$$\text{Let } S = \left\{-12, -7.25, -\sqrt{3}, -\frac{5}{6}, 0, .\overline{45}, \sqrt{9}, \frac{21}{3}, 999\right\}$$

List the elements of *S* that belong to each set.

(c) integers $\left\{-12, 0, \sqrt{9} \text{ (or 3)}, \frac{21}{3} \text{ (or 7)}, 999\right\}$

(d) rational numbers All elements of *S* except $-\sqrt{3}$

Example 1 Identifying Elements of Subsets of the Real Numbers (cont.)

$$\text{Let } S = \left\{-12, -7.25, -\sqrt{3}, -\frac{5}{6}, 0, .\overline{45}, \sqrt{9}, \frac{21}{3}, 999\right\}$$

List the elements of *S* that belong to each set.

(e) irrational numbers $-\sqrt{3}$

(f) real numbers All elements of *S* are real numbers.

R.2 Example 2 Evaluating Exponential Expressions (page 10)

Evaluate each expression and identify the base and the exponent.

(a) 10^3

$$10^3 = \underbrace{10 \cdot 10 \cdot 10}_{3 \text{ factors of } 10} = 64$$

Base: 10

Exponent: 3

(b) $(-3)^4$

$$(-3)^4 = (-3)\cdot(-3)\cdot(-3)\cdot(-3) = 81$$

Base: –3 Exponent: 4

R.2 Example 2 Evaluating Exponential Expressions (cont.)

Evaluate each expression and identify the base and the exponent.

(c) -3^4 $\quad -3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

Base: 3 Exponent: 4

(d) $2 \cdot 5^2$ $\quad 2 \cdot 5^2 = 2 \cdot 5 \cdot 5 = 50$

Base: 5 Exponent: 2

(e) $(2 \cdot 5)^2$ $\quad (2 \cdot 5)^2 = 10^2 = 10 \cdot 10 = 100$

Base: 10 Exponent: 2

R.2 Example 3(a) Using Order of Operations (page 11)

Evaluate

$$6 \div 3 + 2^3 \cdot 5$$

$$6 \div 3 + 2^3 \cdot 5 = 6 \div 3 + 8 \cdot 5$$ Evaluate the exponential

$$= 2 + 8 \cdot 5$$ Divide

$$= 2 + 40$$ Multiply

$$= 42$$ Add

Example 3(b) Using Order of Operations (cont.)

Evaluate

$$(30-5)\cdot 3 \div 15 + 7$$

$$(30-5)\cdot 3 \div 15 + 7 = 25\cdot 3 \div 15 + 7$$

Work within the parenthesis

$$= 75 \div 15 + 7$$

Multiply

$$= 5 + 7$$

Divide

$$= 12$$

Add

R.2 Example 3(c) Using Order of Operations (cont.)

Evaluate

$$\frac{2^4 - 11}{9 + 3 \cdot 2}$$

$$\frac{2^4 - 11}{9 + 3 \cdot 2} = \frac{16 - 11}{9 + 6}$$

Evaluate the exponential

Multiply

$$= \frac{5}{15} = \frac{1}{3}$$

Subtract

Add

Simplify

R.2 Example 3(d) Using Order of Operations (cont.)

Evaluate

$$\frac{-7^2-(-9)}{6(-3)-1(-2)}$$

$$\frac{-7^2-(-9)}{6(-3)-1(-2)}=\frac{-49-(-9)}{6(-3)-1(-2)}$$ Evaluate the exponential

$$=\frac{-49+9}{-18+2}$$ Multiply

$$=\frac{-40}{-16}=\frac{5}{2}$$ Add / Add — Simplify

R.2 Example 4(a) Using Order of Operations (page 11)

Evaluate $6a^2 + 5b - 3c$ using a = –4, b = 3, and c = –6.

$$6a^2 + 5b - 3c = 6(-4)^2 + 5(3) - 3(-6)$$ Substitute

$$= 6(16) + 5(3) - 3(-6)$$ Evaluate the exponential

$$= 96 + 15 + 18$$ Multiply

$$= 129$$ Add

R.2 Example 4(b) Using Order of Operations (cont.)

Evaluate $\dfrac{4b-3(a-1)^2}{c+9}$ using a = –4, b = 3, and c = –6.

$$\frac{4b-3(a-1)^2}{c+9}=\frac{4(3)-3(-4-1)^2}{-6+9}$$

Substitute

$$=\frac{4(3)-3(-5)^2}{-6+9}$$

Subtract

$$=\frac{4(3)-3(25)}{-6+9}$$

Evaluate the exponential

$$=\frac{12-75}{3}=\frac{-63}{3}=-21$$

Multiply, subtract, and simplify

R.2 Example 5(a) Using the Commutative and Associative Properties to Simplify Expressions (page 13)

Simplify $(12 + 2x) + 18$

$(12 + 2x) + 18 = (2x + 12) + 18$ Commutative property

$= 2x + (12 + 18)$ Associative property

$= 2x + 30$

R.2 Example 5(b) Using the Commutative and Associative Properties to Simplify Expressions (cont.)

Simplify $\left(\frac{4}{7}\right)(-35t)$

$$\left(\frac{4}{7}\right)(-35t) = \left(\frac{4}{7}\cdot(-35)\right)t$$

Associative property

$$= -20t$$

R.2 Example 5(c) Using the Commutative and Associative Properties to Simplify Expressions (cont.)

Simplify $(-54s)\left(-\frac{4}{9}\right)$

$$(-54s)\left(-\frac{4}{9}\right) = (s(-54))\left(-\frac{4}{9}\right)$$ Commutative property

$$= s\left(-54 \cdot \left(-\frac{4}{9}\right)\right)$$ Associative property

$$= \left(-54 \cdot \left(-\frac{4}{9}\right)\right)s$$ Commutative property

$$= 24s$$

R.2 Example 6 Using the Distributive Property (page 14)

Rewrite using the distributive property and simplify.

(a) $8(m - 2n) = 8(m - 2n) = 8m - 16n$

(b) $-(-3r + 5s) = -1(-3r + 5s) = -3r - 5s$

R.2 Example 6 Using the Distributive Property (cont.)

Rewrite using the distributive property and simplify.

(c) $\frac{3}{4}\left(\frac{5}{6}p + \frac{1}{2}q - 28\right)$

$$= \frac{3}{4}\left(\frac{5}{6}p + \frac{1}{2}q - 28\right)$$

$$= \frac{3}{4}\left(\frac{5}{6}p\right) + \frac{3}{4}\left(\frac{1}{2}q\right) - \frac{3}{4}(28)$$

$$= \frac{5}{8}p + \frac{3}{8}q - 21$$

R.2 Example 6 Using the Distributive Property (cont.)

Rewrite using the distributive property and simplify.

(d) $22t - 55 = 11(2t) - 11(5) = 11(2t - 5)$

R.2 Example 7 Evaluating Absolute Values (page 15)

Evaluate each expression:

(a) $|-6.85| = 6.85$

(b) $-|50| = -50$

(c) $-\left|-\frac{2}{3}\right| = -\frac{2}{3}$

(d) $|y|$, if $y = \sqrt{2}$

$|\sqrt{2}| = \sqrt{2}$

R.2 Example 8 Measuring Blood Pressure Difference

(page 15)

Find P_d for a patient with a systolic pressure, P, of 146.

$$P_d = |P - 120| = |146 - 120| = |26| = 26$$

R.2 Example 9 Evaluating Absolute Value Expressions

(page 16)

Let *m* = 13 and *n* = –9. Evaluate each expression.

(a) $|3m+5n| = -|3(13)+5(-9)|$

$$= -|39-45| = -|6| = 6$$

(b) $\dfrac{|2m|-3|n|}{|m+n|} = \dfrac{|2(13)|-3|(-9)|}{|13+(-9)|} = \dfrac{|26|-3|-9|}{|4|}$

$$= \frac{26-3(9)}{4} = \frac{26-27}{4} = \frac{1}{4}$$

R.3 Polynomials

Rules for Exponents ▪ Polynomials ▪ Addition and Subtraction ▪ Multiplication ▪ Division

R.3 Example 1 Using the Product Rule (page 22)

Find each product.

(a) $m^6 \cdot m^8 = m^{6+8} = m^{14}$

(b) $\left(-5r^3\right)\left(6r^4\right)(-3r) = (-5)(6)(-3)\cdot\left(r^3r^4r\right)$

Commutative and associative properties

$= 90 \cdot r^{3+4+1}$ Product rule

$= 90r^8$

R.3 Example 2 Using the Power Rules (page 22)

Simplify. Assume all variables represent nonzero real numbers.

(a) $\left(7^3\right)^5 = 7^{3(5)} = 7^{15}$

(b) $\left(2^5 y^3\right)^4 = \left(2^5\right)^4 \left(y^3\right)^4$

$$= 2^{5(4)} y^{3(4)} = 2^{20} y^{12}$$

R.3 Example 2 Using the Power Rules (cont.)

Simplify. Assume all variables represent nonzero real numbers.

(c) $$\left(\frac{4^3}{z^2}\right)^5 = \frac{\left(4^3\right)^5}{\left(z^2\right)^5} = \frac{4^{3(5)}}{z^{2(5)}} = \frac{4^{15}}{z^{10}}$$

(d) $$\left(\frac{-3a^3}{bc^4}\right)^2 = \frac{\left(-3a^3\right)^2}{\left(bc^4\right)^2} = \frac{(-3)^2\left(a^3\right)^2}{b^2\left(c^4\right)^2} = \frac{9a^6}{b^2c^8}$$

R.3 Example 3 Using the Definition of a^0 (page 24)

Evaluate each power.

(a) 8^0

(b) -8^0

(c) $(-8)^0$

(d) $-(-8)^0$

(e) $(-3b^8)^0$

(a) $8^0 = 1$

(b) $-8^0 = -1$

(c) $(-8)^0 = 1$

(d) $-(-8)^0 = -1$

(e) $(-3b^8)^0 = 1, b \neq 0$

R.3 Example 4 Adding and Subtracting Polynomials

(page 24)

Add or subtract.

(a) $$\left(17x^3 - 10x^2 + x\right) + \left(-9x^3 + 10x^2 - 5x\right)$$

$$= (17 - 9)x^3 + (-10 + 10)x^2 + (1 - 5)x$$

$$= 8x^3 - 4x$$

(b) $$\left(-6m^4 - 11m^2 + 21\right) - \left(m^4 - 6m^2 + 35\right)$$

$$= (-6 - 1)m^4 + \left[-11 - (-6)\right]m^2 + (21 - 35)$$

$$= -7m^4 - 5m^2 - 14$$

Example 4 Adding and Subtracting Polynomials (cont.)

Add or subtract.

(c) $\left(10r^3s^6 + 5r^6s^3\right) + \left(25r^3s^6 - 15r^6s^3\right)$

$$= (10 + 25)r^3s^6 + (5 - 15)r^6s^3$$

$$= 35r^3s^6 - 10r^6s^3$$

Example 4 Adding and Subtracting Polynomials (cont.)

Add or subtract.

(d) $6\left(z^2-5z+3\right)-4\left(3z^2-2z+9\right)$

$=6z^2-6(5z)+6(3)-4\left(3z^2\right)-4(-2z)-4(9)$

$=6z^2-30z+18-12z^2+8z-36$

$=-6z^2-22z-18$

R.3 Example 5 Multiplying Polynomials (page 26)

Multiply $(4t-5)(3t^2-2t+7)$

$$
\begin{array}{rrrrl}
 & 3t^2 & -2t & +7 & \\
 & & 4t & -5 & \\
\hline
 & -15t^2 & +10t & -35 & \leftarrow -5(3t^2-2t+7) \\
12t^3 & -8t^2 & +28t & & \leftarrow 4t(3t^2-2t+7) \\
\hline
12t^3 & -23t^2 & +38t & -35 & \text{Add in columns}
\end{array}
$$

Example 6(a) Using FOIL to Multiply Two Binomials (page 27)

Find the product.

$$(7y+3)(4y-5)$$

$$\begin{array}{l} \qquad\quad \text{F} \qquad\qquad \text{O} \qquad\qquad \text{I} \qquad\qquad \text{L} \\ =(7y)(4y)+(7y)(-5)+3(4y)+3(-5) \\ =28y^2-23y-15 \qquad -35y+12y=-23y \end{array}$$

Example 6(b) Using FOIL to Multiply Two Binomials (cont.)

Find the product.

$$(6p+11)(6p-11)$$

F O I L

$$=(6p)(6p)+6p(-11)+11(6p)+11(-11)$$

$$=36p^2-121 \qquad -66p+66p=0$$

Example 6(c) Using FOIL to Multiply Two Binomials (cont.)

Find the product.

$$x^3(2x-5)(2x+5)$$

$$= x^3\left[2x(2x)+2x(5)+(-5)(2x)+(-5)(5)\right] \quad \text{FOIL}$$

$$= x^3\left(4x^2+10x-10x-25\right) = x^3\left(4x^2-25\right)$$

Combine like terms

$$= 4x^5-25x^3 \quad \text{Distributive property}$$

R.3 Example 7 Using the Special Products (page 27)

Find each product.

(a) $(7m-10)(7m+10) = 49m^2 - 100$

(b) $\left(4r^2+9\right)\left(4r^2-9\right) = 16r^4 - 81$

(c) $\left(5x^2-8y^4\right)\left(5x^2+8y^4\right) = 25x^4 - 64y^8$

R.3 Example 7 Using the Special Products (cont.)

Find each product.

(d) $(8z+3)^2 = 64z^2 + 48z + 9$

(e) $\left(5z - 12q^3\right)^2 = 25z^2 - 120zq^3 + 144q^6$

Example 8(a) Multiplying More Complicated Binomials (page 28)

Find the product: $\left[(4x-3)+7y\right]\left[(4x-3)-7y\right]$

$$\left[(4x-3)+7y\right]\left[(4x-3)-7y\right]$$

Product of the sum and difference of two terms

$$=(4x-3)^2-(7y)^2$$

$$=16x^2-24x+9-49y^2$$

R.3 Example 8(b) Multiplying More Complicated Binomials (cont.)

Find the product: $(a-b)^4$

$$(a-b)^4 = (a-b)^2(a-b)^2$$

$$= \left(a^2-2ab+b^2\right)\left(a^2-2ab+b^2\right)$$

$$= a^4-2a^3b+a^2b^2-2a^3b+4a^2b^2$$
$$-2ab^3+a^2b^2-2ab^3+b^4$$

$$= a^4-4a^3b+6a^2b^2-4ab^3+b^4$$

R.3 Example 8(c) Multiplying More Complicated Binomials (cont.)

Find the product: $(s+4t)^3$

$$(s+4t)^3 = (s+4t)^2(s+4t)$$

$$= (s^2+8st+16t^2)(s+4t)$$

$$= s^3+8s^2t+16st^2+4s^2t+32st^2+64t^3$$

$$= s^3+12s^2t+48st^2+64t^3$$

R.3 Example 9 Dividing Polynomials (page 29)

Divide $12n^3 + 11n^2 + 5n - 8$ by $3n + 2$

$$
\begin{array}{r}
4n^2 + n + 1 \\
3n + 2 \overline{\big)\, 12n^3 + 11n^2 + 5n - 8} \\
\underline{12n^3 + 8n^2 } \\
3n^2 + 5n \\
\underline{3n^2 + 2n } \\
3n - 8 \\
\underline{3n + 2} \\
-10
\end{array}
$$

Example 9 Dividing Polynomials (cont.)

$$\left(12n^3+11n^2+5n-8\right)\div\left(3n+2\right)=4n^2+n+1+\frac{-10}{3n+2}$$

R.3 Example 10 Dividing Polynomials with Missing Terms (page 29)

Divide $8x^4 + 12x^2 + 7x - 18$ by $x^2 + 2$

$$
\begin{array}{r}
8x^2 \qquad\qquad -4 \\
x^2 + 0x + 2 \overline{\big)\, 8x^4 + 0x^3 + 12x^2 + 7x - 18} \\
\underline{8x^4 + 0x^3 + 16x^2} \qquad\qquad \\
-4x^2 + 7x - 18 \\
\underline{-4x^2 - 0x - \;\; 8} \\
7x - 10
\end{array}
$$

$$\left(8x^4 + 12x^2 + 7x - 18\right) \div \left(x^2 + 2\right) = 8x^2 - 4 + \frac{7x - 10}{x^2 + 2}$$

R.4 Factoring Polynomials

Factoring Out the Greatest Common Factor ▪ Factoring by Grouping ▪ Factoring Trinomials ▪ Factoring Binomials ▪ Factoring by Substitution

R.4 Example 1 Factoring Out the Greatest Common Factor (page 34)

Factor out the greatest common factor from each polynomial.

(a) $6a^2 - 18a^4 = 6a^2\left(1 - 3a^2\right)$

(b) $14x^3y^2 - 28x^2y^3 + 21x^2y^2$

$= 7x^2y^2(2x - 4y + 3)$

Example 1 Factoring Out the Greatest Common Factor (cont.)

Factor out the greatest common factor from the polynomial.

(c) $24(x-2)^3-16(x-2)^2+6(x-2)$

$$=2(x-2)\left[12(x-2)^2-16(x-2)+1\right]$$

GCF = $2(x-2)$

$$=2(x-2)\left[12\left(x^2-4x+4\right)-8x+16+3\right]$$

$$=2(x-2)\left(12x^2-48x+48-8x+16+3\right)$$

$$=2(x-2)\left(12x^2-56x+67\right)$$

R.4 Example 2(a) Factoring By Grouping (page 35)

Factor by grouping.

$$r^2s + 3r^2 - 5s - 15 = (r^2s + 3r^2) - (5s + 15)$$

$$= r^2(s + 3) - 5(s + 3)$$

$$= (r^2 - 5)(s + 3)$$

R.4 Example 2(b) Factoring By Grouping (page 35)

Factor by grouping.

$$4m^2 - m^2n + 4n - n^2 = (4m^2 - m^2n) + (4n - n^2)$$

$$= m^2(4 - n) + n(4 - n)$$

$$= (m^2 + n)(4 - n)$$

Example 2(c) Factoring By Grouping (page 35)

Factor by grouping.

$$9y^3 - 15y^2 + 6y - 10 = (9y^3 - 15y^2) + (6y - 10)$$

$$= 3y^2(3y - 5) + 2(3y - 5)$$

$$= (3y^2 + 2)(3y - 5)$$

R.4 Example 3(a) Factoring Trinomials (page 36)

Factor $5z^2 + 4z - 12$, if possible.

The positive factors of 5 are 5 and 1.

The factors of –12 are –12 and 1, 12 and – 1, –6 and 2, 6 and –2, –4 and 3, or 4 and –3.

Example 3(a) Factoring Trinomials (cont.)

Factor $5z^2+4z-12$.

Try different combinations:

$(5z-12)(z+1)=5z^2-7z-12$ incorrect

$(5z+4)(z-3)=5z^2-11z-12$ incorrect

$(5z-6)(z+2)=5z^2+4z-12$ correct

$$5z^2+4z-12=(5z-6)(z+2)$$

R.4 Example 3(b) Factoring Trinomials (page 36)

Factor $12t^2 - 5t - 3$, if possible.

The positive factors of 12 are 12 and 1, 6 and 2, or 4 and 3.

The factors of –3 are –3 and 1 or 3 and –1.

R.4 Example 3(b) Factoring Trinomials (cont.)

Factor $12t^2 - 5t - 3$.

Try different combinations:

$$(2t + 3)(6t - 1) = 12t^2 + 16t - 3$$ incorrect

$$(12t - 3)(t + 1) = 12t^2 + 9t - 3$$ incorrect

$$(4t - 3)(3t + 1) = 12t^2 - 5t - 3$$ correct

$$12t^2 - 5t - 3 = (4t - 3)(3t + 1)$$

Example 3(c) Factoring Trinomials (page 36)

Factor $3x^2 - 15x + 16$, if possible.

The positive factors of 3 are 3 and 1.

The negative factors of 16 are –16 and – 1, – 8 and – 2, or – 4 and –4.

R.4 Example 3(c) Factoring Trinomials (page 36)

Factor $3x^2 - 15x + 16$.

Try different combinations:

$(3x-16)(x-1) = 3x^2 - 19x + 16$ incorrect

$(3x-1)(x-16) = 3x^2 - 49x + 16$ incorrect

$(3x-8)(x-2) = 3x^2 - 14x + 16$ incorrect

$(3x-2)(x-8) = 3x^2 - 26x + 16$ incorrect

$(3x-4)(x-4) = 3x^2 - 16x + 16$ incorrect

$3x^2 - 15x + 16$ is prime.

R.4 Example 3(d) Factoring Trinomials (page 36)

Factor $24x^2 + 42x + 15$, if possible.

Factor out the GCF, 3, first:

$$24x^2 + 42x + 15 = 3\left(8x^2 + 14x + 5\right)$$

The positive factors of 8 are 8 and 1 or 4 and 2.

The positive factors of 5 are 5 and 1.

R.4 Example 3(d) Factoring Trinomials (page 36)

Factor $24x^2 + 42x + 15$.

Try different combinations:

$$(2x+5)(4x+1) = 8x^2 + 22x + 5$$ incorrect

$$(8x+5)(x+1) = 8x^2 + 13x + 5$$ incorrect

$$(4x+5)(2x+1) = 8x^2 + 14x + 5$$ correct

$$24x^2 + 42x + 15 = 3\left(8x^2 + 14x + 5\right) = 3(4x+5)(2x+1)$$

R.4 Example 4 Factoring Perfect Square Trinomials (page 37)

Factor each trinomial:

(a) $49x^2 + 28xy + 4y^2 = (7x + 2y)^2$

(b) $81a^2b^2 - 90ab + 25 = (9ab - 5)^2$

R.4 Example 5 Factoring Differences of Squares (page 38)

Factor each trinomial:

(a) $64r^2 - 49 = (8r - 7)(8r + 7)$

(b) $169u^6 - 144v^4 = \left(13u^3 + 12v^2\right)\left(13u^3 - 12v^2\right)$

(c) $(2c - 3d)^2 - 16f^2 = (2c - 3d + 4f)(2c - 3d - 4f)$

R.4 Example 5 Factoring Differences of Squares (cont.)

Factor the trinomial:

(d) $x^2 + 18x + 81 - 25y^2$

$$= \left(x^2 + 18x + 81\right) - 25y^2$$

$$= (x+9)^2 - 25y^2$$

$$= (x+9+5y)(x+9-5y)$$

Example 5 Factoring Differences of Squares (cont.)

Factor the trinomial:

(e) $4x^2 - y^2 - 10y - 25$

$$= 4x^2 - \left(y^2 + 10y + 25\right)$$

$$= 4x^2 - (y + 5)^2$$

$$= \left(2x + (y + 5)\right)\left(2x - (y + 5)\right)$$

$$= (2x + y + 5)(2x - y - 5)$$

R.4 Example 6 Factoring Sums or Differences of Cubes (page 39)

Factor each polynomial:

(a) $t^3 + 1000 = t^3 + 10^3 = (t + 10)(t^2 - 10x + 10^2)$

$= (t + 10)(t^2 - 10x + 100)$

(b) $r^3 - 8s^3 = r^3 - (2s)^3$

$= (r - 2s)\left[r^2 - r(2s) + (2s)^2\right]$

$= (r - 2s)(r^2 - 2rs + 4s^2)$

R.4 Example 6 Factoring Sums or Differences of Cubes (cont.)

Factor each polynomial:

(c) $125u^9 - 216v^{12}$

$$= \left(5u^3\right)^3 - \left(6v^4\right)^3$$

$$= \left(5u^3 - 6v^4\right)^3 \left[\left(5u^3\right)^2 + \left(5u^3\right)\left(6v^4\right) + \left(6v^4\right)^2\right]$$

$$= \left(5u^3 - 6v^4\right)^3 \left(25u^6 + 30u^3v^4 + 36v^8\right)$$

R.4 Example 7(a) Factoring by Substitution (page 40)

Factor $15m^4 - m^2 - 6$

Replace m^2 with u: $15m^4 - m^2 - 6 = 15u^2 - u - 6$

Factor using FOIL: $15u^2 - u - 6 = (3u - 2)(5u + 3)$

Replace u with m^2:

$$(3u - 2)(5u + 3) = \left(3m^2 - 2\right)\left(5m^2 + 3\right)$$

$$15m^4 - m^2 - 6 = \left(3m^2 - 2\right)\left(5m^2 + 3\right)$$

Example 7(b) Factoring by Substitution (page 40)

Factor $8(3x+1)^2+10(3x+1)-25$

Replace (3*x* + 1) with *u*:

$$8(3x+1)^2+10(3x+1)-25=8u^2+10u-25$$

Factor using FOIL: $8u^2+10u-25=(2u+5)(4u-5)$

Replace *u* with (3*x* + 1), then simplify:

$$\begin{aligned}(2u+5)(4u-5)&=\left[2(3x+1)+5\right]\left[4(3x+1)-5\right]\\&=(6x+2+5)(12x+4-5)\\&=(6x+7)(12x-1)\end{aligned}$$

$$8(3x+1)^2+10(3x+1)-25=(6x+7)(12x-1)$$

R.4 Example 7(c) Factoring by Substitution (page 40)

Factor $(3x+1)^3 - 27$

Replace (3*x* + 1) with *u*: $(3x+1)^3 - 27 = u^3 - 27$

Write as the difference of cubes, then factor:

$$u^3 - 27 = u^3 - 3^3 = (u-3)\left(u^2 + 3u + 9\right)$$

Replace *u* with (3*x* + 1), then simplify:

$$(u-3)\left(u^2+3u+9\right) = \left[(3x+1)-3\right]\left[(3x+1)^2 + 3(3x+1) + 9\right]$$

$$= (3x-2)\left[9x^2 + 6x + 1 + 9x + 3 + 9\right]$$

$$= (3x-2)\left(9x^2 + 15x + 13\right)$$

$$(3x+1)^3 - 27 = (3x-2)\left(9x^2 + 15x + 13\right)$$