Factoring Polynomials

Factoring Out the Greatest Common Factor ▪ Factoring by Grouping ▪ Factoring Trinomials ▪ Factoring Binomials ▪ Factoring by Substitution

The process of finding polynomials whose product equals a given polynomial is called **factoring.** In this book, we consider only integer coefficients when factoring polynomials. For example, since $4x + 12 = 4(x + 3)$, both 4 and $x + 3$ are called **factors** of $4x + 12$. Also, $4(x + 3)$ is called the **factored form** of $4x + 12$. A polynomial with variable terms that cannot be written as a product of two polynomials of lower degree is a **prime polynomial.** A polynomial is **factored completely** when it is written as a product of prime polynomials.

Factoring Out the Greatest Common Factor

To factor $6x^2y^3 + 9xy^4 + 18y^5$, we look for a monomial that is the greatest common factor (GCF) of each term.

$$6x^2y^3 + 9xy^4 + 18y^5 = 3y^3(2x^2) + 3y^3(3xy) + 3y^3(6y^2) \quad \text{GCF} = 3y^3$$

$$= 3y^3(2x^2 + 3xy + 6y^2) \quad \text{Distributive property (Section R.2)}$$

▶ EXAMPLE 1 FACTORING OUT THE GREATEST COMMON FACTOR

Factor out the greatest common factor from each polynomial.

(a) $9y^5 + y^2$ **(b)** $6x^2t + 8xt + 12t$

(c) $14(m + 1)^3 - 28(m + 1)^2 - 7(m + 1)$

Solution

(a) $9y^5 + y^2 = y^2(9y^3) + y^2(1)$ GCF $= y^2$

$= y^2(9y^3 + 1)$ Distributive property

Remember to include the 1.

Original polynomial

To *check*, multiply out the factored form: $y^2(9y^3 + 1) = 9y^5 + y^2$.

(b) $6x^2t + 8xt + 12t = 2t(3x^2 + 4x + 6)$ GCF $= 2t$

Check: $2t(3x^2 + 4x + 6) = 6x^2t + 8xt + 12t$

(c) $14(m + 1)^3 - 28(m + 1)^2 - 7(m + 1)$

$= 7(m + 1)[2(m + 1)^2 - 4(m + 1) - 1]$ GCF $= 7(m + 1)$

$= 7(m + 1)[2(m^2 + 2m + 1) - 4m - 4 - 1]$ Square $m + 1$ (Section R.3); distributive property

Remember the middle term.

$= 7(m + 1)(2m^2 + 4m + 2 - 4m - 4 - 1)$ Distributive property

Factoring by Grouping

When a polynomial has more than three terms, it can sometimes be factored using **factoring by grouping.** For example, to factor

$$ax + ay + 6x + 6y,$$

group the terms so that each group has a common factor.

Terms with common factor a — Terms with common factor 6

$$ax + ay + 6x + 6y = (ax + ay) + (6x + 6y)$$

$$= a(x + y) + 6(x + y) \quad \text{Factor each group.}$$

$$= (x + y)(a + 6) \quad \text{Factor out } x + y.$$

It is not always obvious which terms should be grouped. Experience and repeated trials are the most reliable tools when factoring.

▶ EXAMPLE 2 FACTORING BY GROUPING

Factor each polynomial by grouping.

(a) $mp^2 + 7m + 3p^2 + 21$ **(b)** $2y^2 + az - 2z - ay^2$

(c) $4x^3 + 2x^2 - 2x - 1$

Solution

(a) $mp^2 + 7m + 3p^2 + 21 = (mp^2 + 7m) + (3p^2 + 21)$ Group the terms.

$= m(p^2 + 7) + 3(p^2 + 7)$ Factor each group.

$= (p^2 + 7)(m + 3)$ $p^2 + 7$ is a common factor.

Check: $(p^2 + 7)(m + 3) = mp^2 + 3p^2 + 7m + 21$ FOIL (Section R.3)

$= mp^2 + 7m + 3p^2 + 21$ Commutative property (Section R.2)

(b) $2y^2 + az - 2z - ay^2 = 2y^2 - 2z - ay^2 + az$ Rearrange the terms.

$= (2y^2 - 2z) + (-ay^2 + az)$ Group the terms.

$= 2(y^2 - z) + a(-y^2 + z)$ Factor each group.

Be careful with signs here.

$= 2(y^2 - z) - a(y^2 - z)$ Factor out $-a$ instead of a.

$= (y^2 - z)(2 - a)$ Factor out $y^2 - z$.

Check by multiplying.

Factoring Trinomials As shown here, factoring is the opposite of multiplication.

Multiplication →

$$(2x + 1)(3x - 4) = 6x^2 - 5x - 4$$

← Factoring

Since the product of two binomials is usually a trinomial, we can expect factorable trinomials (that have terms with no common factor) to have two binomial factors. Thus, factoring trinomials requires using FOIL in reverse.

▶ EXAMPLE 3 FACTORING TRINOMIALS

Factor each trinomial, if possible.

(a) $4y^2 - 11y + 6$ **(b)** $6p^2 - 7p - 5$

(c) $2x^2 + 13x - 18$ **(d)** $16y^3 + 24y^2 - 16y$

Solution

(a) To factor this polynomial, we must find integers a, b, c, and d such that

$$4y^2 - 11y + 6 = (ay + b)(cy + d). \quad \text{FOIL}$$

Using FOIL, we see that $ac = 4$ and $bd = 6$. The positive factors of 4 are 4 and 1 or 2 and 2. Since the middle term is negative, we consider only negative factors of 6. The possibilities are -2 and -3 or -1 and -6. Now we try various arrangements of these factors until we find one that gives the correct coefficient of y.

$$(2y - 1)(2y - 6) = 4y^2 - 14y + 6 \quad \text{Incorrect}$$
$$(2y - 2)(2y - 3) = 4y^2 - 10y + 6 \quad \text{Incorrect}$$
$$(y - 2)(4y - 3) = 4y^2 - 11y + 6 \quad \text{Correct}$$

Therefore, $4y^2 - 11y + 6 = (y - 2)(4y - 3)$.

Check: $(y - 2)(4y - 3) = 4y^2 - 3y - 8y + 6$ FOIL
$= 4y^2 - 11y + 6$ Original polynomial

(b) Again, we try various possibilities to factor $6p^2 - 7p - 5$. The positive factors of 6 could be 2 and 3 or 1 and 6. As factors of -5 we have only -1 and 5 or -5 and 1.

$$(2p - 5)(3p + 1) = 6p^2 - 13p - 5 \quad \text{Incorrect}$$
$$(3p - 5)(2p + 1) = 6p^2 - 7p - 5 \quad \text{Correct}$$

Therefore, $6p^2 - 7p - 5 = (3p - 5)(2p + 1)$. *Check.*

(c) If we try to factor $2x^2 + 13x - 18$ as above, we find that none of the pairs of factors gives the correct coefficient of x. For example,

$$(2x + 9)(x - 2) = 2x^2 + 5x - 18 \quad \text{Incorrect}$$
$$(2x - 3)(x + 6) = 2x^2 + 9x - 18 \quad \text{Incorrect}$$
$$(2x - 1)(x + 18) = 2x^2 + 35x - 18. \quad \text{Incorrect}$$

Additional trials are also unsuccessful. Thus, this trinomial cannot be factored with integer coefficients and is prime.

(d) $16y^3 + 24y^2 - 16y = 8y(2y^2 + 3y - 2)$ Factor out the GCF, $8y$.
$= 8y(2y - 1)(y + 2)$ Factor the trinomial.

Remember the common factor.

NOW TRY EXERCISES 25, 27, 29, AND 31. ◀

▶ **Note** In Example 3, we chose positive factors of the positive first term. We could have used two negative factors, but the work is easier if positive factors are used.

Each of the special patterns for multiplication given in **Section R.3** can be used in reverse to get a pattern for factoring. Perfect square trinomials can be factored as follows.

FACTORING PERFECT SQUARE TRINOMIALS

$$x^2 + 2xy + y^2 = (x + y)^2$$
$$x^2 - 2xy + y^2 = (x - y)^2$$

▶ EXAMPLE 4 FACTORING PERFECT SQUARE TRINOMIALS

Factor each trinomial.

(a) $16p^2 - 40pq + 25q^2$ **(b)** $36x^2y^2 + 84xy + 49$

Solution

(a) Since $16p^2 = (4p)^2$ and $25q^2 = (5q)^2$, we use the second pattern shown in the box with $4p$ replacing x and $5q$ replacing y.

$$16p^2 - 40pq + 25q^2 = (4p)^2 - 2(4p)(5q) + (5q)^2$$
$$= (4p - 5q)^2$$

Make sure that the middle term of the trinomial being factored, $-40pq$ here, is twice the product of the two terms in the binomial $4p - 5q$.

$$-40pq = 2(4p)(-5q)$$

Thus, $16p^2 - 40pq + 25q^2 = (4p - 5q)^2$.

Check by squaring $4p - 5q$: $(4p - 5q)^2 = 16p^2 - 40pq + 25q^2$.

(b) $36x^2y^2 + 84xy + 49 = (6xy + 7)^2$, since $2(6xy)(7) = 84xy$.

Check by squaring $6xy + 7$: $(6xy + 7)^2 = 36x^2y^2 + 84xy + 49$.

NOW TRY EXERCISES 41 AND 45.

Factoring Binomials Check first to see whether the terms of a binomial have a common factor. If so, factor it out. The binomial may also fit one of the following patterns.

FACTORING BINOMIALS

Difference of Squares	$x^2 - y^2 = (x + y)(x - y)$
Difference of Cubes	$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
Sum of Cubes	$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Caution *There is no factoring pattern for a sum of squares in the real number system.* That is, $x^2 + y^2 \neq (x + y)^2$, for real numbers x and y.

EXAMPLE 5 FACTORING DIFFERENCES OF SQUARES

Factor each polynomial.

(a) $4m^2 - 9$ **(b)** $256k^4 - 625m^4$ **(c)** $(a + 2b)^2 - 4c^2$

(d) $x^2 - 6x + 9 - y^4$ **(e)** $y^2 - x^2 + 6x - 9$

Solution

(a) $4m^2 - 9 = (2m)^2 - 3^2$ Write as the difference of squares.

$= (2m + 3)(2m - 3)$ Factor.

Check by multiplying.

(b) $256k^4 - 625m^4 = (16k^2)^2 - (25m^2)^2$ Write as the difference of squares.

Don't stop here. $= (16k^2 + 25m^2)(16k^2 - 25m^2)$ Factor.

$= (16k^2 + 25m^2)(4k + 5m)(4k - 5m)$ Factor $16k^2 - 25m^2$.

Check: $(16k^2 + 25m^2)(4k + 5m)(4k - 5m)$

$= (16k^2 + 25m^2)(16k^2 - 25m^2)$ Multiply the last two factors.

$= 256k^4 - 625m^4$ Original polynomial

(c) $(a + 2b)^2 - 4c^2 = (a + 2b)^2 - (2c)^2$ Write as the difference of squares.

$= [(a + 2b) + 2c][(a + 2b) - 2c]$ Factor.

$= (a + 2b + 2c)(a + 2b - 2c)$

Check by multiplying.

(d) $x^2 - 6x + 9 - y^4 = (x^2 - 6x + 9) - y^4$ Group terms.

$= (x - 3)^2 - y^4$ Factor the trinomial.

$= (x - 3)^2 - (y^2)^2$ Write as the difference of squares.

$= [(x - 3) + y^2][(x - 3) - y^2]$ Factor.

$= (x - 3 + y^2)(x - 3 - y^2)$

Check by multiplying.

(e) $y^2 - x^2 + 6x - 9 = y^2 - (x^2 - 6x + 9)$ Factor out the negative and group the last three terms.

Be careful with signs.

$= y^2 - (x - 3)^2$ Write as the difference of squares.

$= [y - (x - 3)][y + (x - 3)]$ Factor.

$= (y - x + 3)(y + x - 3)$ Distributive property

Check by multiplying.

NOW TRY EXERCISES 51, 55, 57, AND 59.

EXAMPLE 6 FACTORING SUMS OR DIFFERENCES OF CUBES

Factor each polynomial.

(a) $x^3 + 27$ **(b)** $m^3 - 64n^3$ **(c)** $8q^6 + 125p^9$

Solution

(a) $x^3 + 27$ $x^3 + 3^3$ Write as a sum of cubes.

$(x + 3)(x^2 - 3x + 3^2)$ Factor.

$= (x + 3)(x^2 - 3x + 9)$ Apply the exponent.

(b) ${}^3 - 64n^3 = m^3 - (4n)^3$ Write as a difference of cubes.

$= (m - 4n)[m^2 + m(4n) + (4n)^2]$ Factor.

$= (m - 4n)(m^2 + 4mn + 16n^2)$ Simplify.

(c) $8q^6 + 125p^9 = (2q^2)^3 + (5p^3)^3$ Write as a sum of cubes.

$= (2q^2 + 5p^3)[(2q^2)^2 - 2q^2(5p^3) + (5p^3)^2]$ Factor.

$= (2q^2 + 5p^3)(4q^4 - 10q^2p^3 + 25p^6)$ Simplify.

NOW TRY EXERCISES 61, 63, AND 65.

Factoring by Substitution Sometimes a polynomial can be more easily factored by substituting one expression for another.

▶ EXAMPLE 7 FACTORING BY SUBSTITUTION

Factor each polynomial.

(a) $6z^4 - 13z^2 - 5$ **(b)** $10(2a - 1)^2 - 19(2a - 1) - 15$

(c) $(2a - 1)^3 + 8$

Solution

(a) Replace z^2 with u, so $u^2 = (z^2)^2 = z^4$.

$6z^4 - 13z^2 - 5 = 6u^2 - 13u - 5$

$= (2u - 5)(3u + 1)$ Use FOIL to factor.

$= (2z^2 - 5)(3z^2 + 1)$ Replace u with z^2.

Remember to make the final substitution.

(Some students prefer to factor this type of trinomial directly using trial and error with FOIL.)

(b) $10(2a - 1)^2 - 19(2a - 1) - 15$

$= 10u^2 - 19u - 15$ Replace $2a - 1$ with u.

$= (5u + 3)(2u - 5)$ Factor.

$= [5(2a - 1) + 3][2(2a - 1) - 5]$ Let $u = 2a - 1$.

$= (10a - 5 + 3)(4a - 2 - 5)$ Distributive property

$= (10a - 2)(4a - 7)$ Add.

$= 2(5a - 1)(4a - 7)$ Factor out the common factor.

Don't stop here. Replace u with $2a - 1$.

(c) $(2a - 1)^3 + 8 = u^3 + 8$ Let $2a - 1 = u$.

$= u^3 + 2^3$ Write as a sum of cubes.

$= (u + 2)(u^2 - 2u + 4)$ Factor.

$= [(2a - 1) + 2][(2a - 1)^2 - 2(2a - 1) + 4]$ Let $u = 2a - 1$.

$= (2a + 1)(4a^2 - 4a + 1 - 4a + 2 + 4)$ Add; multiply.

$= (2a + 1)(4a^2 - 8a + 7)$ Combine like terms.

NOW TRY EXERCISES 79, 81, AND 97. ◀

R.4 Exercises

Factor out the greatest common factor from each polynomial. See Examples 1 and 2.

1. $12m + 60$ **2.** $15r - 27$ **3.** $8k^3 + 24k$

4. $9z^4 + 81z$ **5.** $xy - 5xy^2$ **6.** $5h^2j + hj$

7. $-4p^3q^4 - 2p^2q^5$ **8.** $-3z^5w^2 - 18z^3w^4$

9. $4k^2m^3 + 8k^4m^3 - 12k^2m^4$ **10.** $28r^4s^2 + 7r^3s - 35r^4s^3$

11. $2(a + b) + 4m(a + b)$ **12.** $4(y - 2)^2 + 3(y - 2)$

13. $(5r - 6)(r + 3) - (2r - 1)(r + 3)$ **14.** $(3z + 2)(z + 4) - (z + 6)(z + 4)$

15. $2(m - 1) - 3(m - 1)^2 + 2(m - 1)^3$ **16.** $5(a + 3)^3 - 2(a + 3) + (a + 3)^2$

17. *Concept Check* When directed to completely factor the polynomial $4x^2y^5 - 8xy^3$, a student wrote $2xy^3(2xy^2 - 4)$. When the teacher did not give him full credit, he complained because when his answer is multiplied out, the result is the original polynomial. Give the correct answer.

Factor each polynomial by grouping. See Example 2.

18. $10ab - 6b + 35a - 21$ **19.** $6st + 9t - 10s - 15$

20. $15 - 5m^2 - 3r^2 + m^2r^2$ **21.** $2m^4 + 6 - am^4 - 3a$

22. $20z^2 - 8x + 5pz^2 - 2px$ **23.** $p^2q^2 - 10 - 2q^2 + 5p^2$

24. *Concept Check* Layla factored $16a^2 - 40a - 6a + 15$ by grouping and obtained $(8a - 3)(2a - 5)$. Jamal factored the same polynomial and gave an answer of $(3 - 8a)(5 - 2a)$. Which answer is correct?

Factor each trinomial, if possible. See Examples 3 and 4.

25. $6a^2 - 11a + 4$ **26.** $8h^2 - 2h - 21$ **27.** $3m^2 + 14m + 8$

28. $9y^2 - 18y + 8$ **29.** $15p^2 + 24p + 8$ **30.** $9x^2 + 4x - 2$

31. $12a^3 + 10a^2 - 42a$ **32.** $36x^3 + 18x^2 - 4x$ **33.** $6k^2 + 5kp - 6p^2$

34. $14m^2 + 11mr - 15r^2$ **35.** $5a^2 - 7ab - 6b^2$ **36.** $12s^2 + 11st - 5t^2$

37. $12x^2 - xy - y^2$ **38.** $30a^2 + am - m^2$ **39.** $24a^4 + 10a^3b - 4a^2b^2$

40. $18x^5 + 15x^4z - 75x^3z^2$ **41.** $9m^2 - 12m + 4$ **42.** $16p^2 - 40p + 25$

43. $32a^2 + 48ab + 18b^2$ **44.** $20p^2 - 100pq + 125q^2$

45. $4x^2y^2 + 28xy + 49$ **46.** $9m^2n^2 + 12mn + 4$

47. $(a - 3b)^2 - 6(a - 3b) + 9$ **48.** $(2p + q)^2 - 10(2p + q) + 25$

49. *Concept Check* Match each polynomial in Column I with its factored form in Column II.

I	II
(a) $x^2 + 10xy + 25y^2$	**A.** $(x + 5y)(x - 5y)$
(b) $x^2 - 10xy + 25y^2$	**B.** $(x + 5y)^2$
(c) $x^2 - 25y^2$	**C.** $(x - 5y)^2$
(d) $25y^2 - x^2$	**D.** $(5y + x)(5y - x)$

50. *Concept Check* Match each polynomial in Column I with its factored form in Column II.

I	II
(a) $8x^3 - 27$	**A.** $(3 - 2x)(9 + 6x + 4x^2)$
(b) $8x^3 + 27$	**B.** $(2x - 3)(4x^2 + 6x + 9)$
(c) $27 - 8x^3$	**C.** $(2x + 3)(4x^2 - 6x + 9)$

Factor each polynomial. See Examples 5 and 6. (In Exercises 53 and 54, factor over the rational numbers.)

51. $9a^2 - 16$ **52.** $16q^2 - 25$ **53.** $36x^2 - \frac{16}{25}$

54. $100y^2 - \frac{4}{49}$ **55.** $25s^4 - 9t^2$ **56.** $36z^2 - 81y^4$

57. $(a + b)^2 - 16$ **58.** $(p - 2q)^2 - 100$ **59.** $p^4 - 625$

60. $m^4 - 81$ **61.** $8 - a^3$ **62.** $r^3 + 27$

63. $125x^3 - 27$ **64.** $8m^3 - 27n^3$ **65.** $27y^9 + 125z^6$

66. $27z^3 + 729y^3$ **67.** $(r + 6)^3 - 216$ **68.** $(b + 3)^3 - 27$

69. $27 - (m + 2n)^3$ **70.** $125 - (4a - b)^3$

71. *Concept Check* Which of the following is the correct complete factorization of $x^4 - 1$?

A. $(x^2 - 1)(x^2 + 1)$ **B.** $(x^2 + 1)(x + 1)(x - 1)$
C. $(x^2 - 1)^2$ **D.** $(x - 1)^2(x + 1)^2$

72. *Concept Check* Which of the following is the correct factorization of $x^3 + 8$?

A. $(x + 2)^3$ **B.** $(x + 2)(x^2 + 2x + 4)$
C. $(x + 2)(x^2 - 2x + 4)$ **D.** $(x + 2)(x^2 - 4x + 4)$

RELATING CONCEPTS

For individual or collaborative investigation
(Exercises 73–78)

The polynomial $x^6 - 1$ can be considered either a difference of squares or a difference of cubes. ***Work Exercises 73–78 in order,*** *to connect the results obtained when two different methods of factoring are used.*

73. Factor $x^6 - 1$ by first factoring as the difference of squares, and then factor further by using the patterns for the sum of cubes and the difference of cubes.

74. Factor $x^6 - 1$ by first factoring as the difference of cubes, and then factor further by using the pattern for the difference of squares.

75. Compare your answers in Exercises 73 and 74. Based on these results, what is the factorization of $x^4 + x^2 + 1$?

76. The polynomial $x^4 + x^2 + 1$ cannot be factored using the methods described in this section. However, there is a technique that allows us to factor it, as shown here. Supply the reason that each step is valid.

$$\begin{aligned} x^4 + x^2 + 1 &= x^4 + 2x^2 + 1 - x^2 \\ &= (x^4 + 2x^2 + 1) - x^2 \\ &= (x^2 + 1)^2 - x^2 \\ &= (x^2 + 1 - x)(x^2 + 1 + x) \\ &= (x^2 - x + 1)(x^2 + x + 1) \end{aligned}$$

77. Compare your answer in Exercise 75 with the final line in Exercise 76. What do you notice?

78. Factor $x^8 + x^4 + 1$ using the technique outlined in Exercise 76.

Factor each polynomial by substitution. See Example 7.

79. $m^4 - 3m^2 - 10$
80. $a^4 - 2a^2 - 48$
81. $7(3k - 1)^2 + 26(3k - 1) - 8$
82. $6(4z - 3)^2 + 7(4z - 3) - 3$
83. $9(a - 4)^2 + 30(a - 4) + 25$
84. $20(4 - p)^2 - 3(4 - p) - 2$

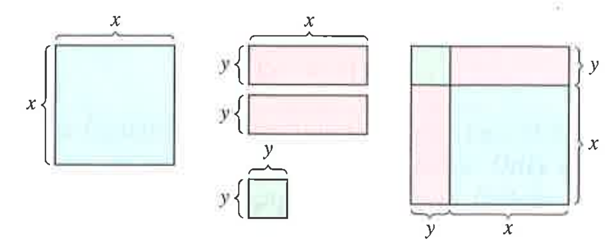
Factor by any method. See Examples 1–7. (Factor Exercise 99 over the rational numbers.)

85. $4b^2 + 4bc + c^2 - 16$
86. $(2y - 1)^2 - 4(2y - 1) + 4$
87. $x^2 + xy - 5x - 5y$
88. $8r^2 - 3rs + 10s^2$
89. $p^4(m - 2n) + q(m - 2n)$
90. $36a^2 + 60a + 25$
91. $4z^2 + 28z + 49$
92. $6p^4 + 7p^2 - 3$

93. $1000x^3 + 343y^3$
94. $b^2 + 8b + 16 - a^2$
95. $125m^6 - 216$
96. $q^2 + 6q + 9 - p^2$
97. $64 + (3x + 2)^3$
98. $216p^3 + 125q^3$
99. $\frac{4}{25}x^2 - 49y^2$
100. $100r^2 - 169s^2$
101. $144z^2 + 121$
102. $(3a + 5)^2 - 18(3a + 5) + 81$
103. $(x + y)^2 - (x - y)^2$
104. $4z^4 - 7z^2 - 15$

105. Are there any conditions under which a sum of squares can be factored? If so, give an example.

106. *Geometric Modeling* Explain how the figures give geometric interpretation to the formula $x^2 + 2xy + y^2 = (x + y)^2$.



Concept Check Find all values of b or c that will make the polynomial a perfect square trinomial.

107. $4z^2 + bz + 81$
108. $9p^2 + bp + 25$
109. $100r^2 - 60r + c$
110. $49x^2 + 70x + c$

R.5 Rational Expressions

Rational Expressions ▪ Lowest Terms of a Rational Expression ▪ Multiplication and Division ▪ Addition and Subtraction ▪ Complex Fractions

Rational Expressions

The quotient of two polynomials P and Q, with $Q \neq 0$, is called a **rational expression.**

$$\frac{x + 6}{x + 2}, \quad \frac{(x + 6)(x + 4)}{(x + 2)(x + 4)}, \quad \frac{2p^2 + 7p - 4}{5p^2 + 20p} \qquad \text{Rational expressions}$$

The **domain** of a rational expression is the set of real numbers for which the expression is defined. Because the denominator of a fraction cannot be 0, the domain consists of all real numbers except those that make the denominator 0. We find these numbers by setting the denominator equal to 0 and solving the resulting equation. For example, in the rational expression

$$\frac{x + 6}{x + 2},$$

the solution to the equation $x + 2 = 0$ is excluded from the domain. Since this solution is -2, the domain is the set of all real numbers x not equal to -2, written $\{x \mid x \neq -2\}$.