R.1 Sets

Basic Definitions ▪ Operations on Sets

Basic Definitions

We think of a **set** as a collection of objects. The objects that belong to a set are called the **elements** or **members** of the set. In algebra, the elements of a set are usually numbers. Sets are commonly written using **set braces,** $\{\ \}$. For example, the set containing the elements 1, 2, 3, and 4 is written

$$\{1, 2, 3, 4\}.$$

Since the order in which the elements are listed is not important, this same set can also be written as $\{4, 3, 2, 1\}$ or with any other arrangement of the four numbers.

To show that 4 is an element of the set $\{1, 2, 3, 4\}$, we use the symbol $\in$ and write

$$4 \in \{1, 2, 3, 4\}.$$

Also, $2 \in \{1, 2, 3, 4\}$. To show that 5 is *not* an element of this set, we place a slash through the symbol:

$$5 \notin \{1, 2, 3, 4\}.$$

NOW TRY EXERCISES 17 AND 19. ◀

It is customary to name sets with capital letters. If S is used to name the set above, then

$$S = \{1, 2, 3, 4\}.$$

Set S was written by listing its elements. It is sometimes easier to describe a set in words. For example, set S might be described as "the set containing the first four counting numbers." In this example, the notation $\{1, 2, 3, 4\}$, with the elements listed between set braces, is briefer than the verbal description. However, the set F, consisting of all fractions between 0 and 1, could not be described by listing its elements. (Try it.)

Set F is an example of an **infinite set,** one that has an unending list of distinct elements. A **finite set** is one that has a limited number of elements. Some infinite sets, unlike F, can be described by listing. For example, the set of numbers used for counting, called the **natural numbers** or the **counting numbers,** can be written as

$$N = \{1, 2, 3, 4, \ldots\}, \quad \text{Natural (counting) numbers}$$

where the three dots (*ellipsis points*) show that the list of elements of the set continues according to the established pattern.

NOW TRY EXERCISES 9 AND 11. ◀

Sets are often written using a variable. For example,

$$\{x \mid x \text{ is a natural number between 2 and 7}\}$$

(read "the set of all elements x such that x is a natural number between 2 and 7") represents the set $\{3, 4, 5, 6\}$. The numbers 2 and 7 are *not* between 2 and 7. The notation used here, $\{x \mid x$ is a natural number between 2 and 7$\}$, is called **set-builder notation.**

▶ EXAMPLE 1 LISTING THE ELEMENTS OF A SET

Write the elements belonging to each set.

(a) $\{x \mid x$ is a counting number less than 5$\}$

(b) $\{x \mid x$ is a state that borders Florida$\}$

Solution

(a) The counting numbers less than 5 make up the set $\{1, 2, 3, 4\}$.

(b) The states bordering Florida make up the set {Alabama, Georgia}.

NOW TRY EXERCISE 7. ◀

When discussing a particular situation or problem, we can usually identify a **universal set** (whether expressed or implied) that contains all the elements appearing in any set used in the given problem. The letter U is used to represent the universal set.

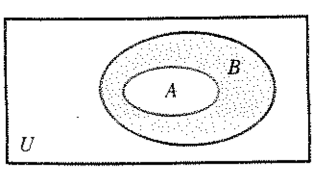
At the other extreme from the universal set is the **null set,** or **empty set,** the set containing no elements. The set of all people twelve feet tall is an example of the null set. We write the null set in either of two ways: using the special symbol $\emptyset$ or else writing set braces enclosing no elements, $\{\ \}$.

▶ **Caution** Do not combine these symbols; $\{\emptyset\}$ is *not* the null set.

Every element of the set $S = \{1, 2, 3, 4\}$ is a natural number. Because of this, set S is a *subset* of the set N of natural numbers, written $S \subseteq N$. By definition, set A is a **subset** of set B if every element of set A is also an element of set B. For example, if $A = \{2, 5, 9\}$ and $B = \{2, 3, 5, 6, 9, 10\}$, then $A \subseteq B$. However, there are some elements of B that are not in A, so B is not a subset of A, written $B \not\subseteq A$. By the definition, every set is a subset of itself. Also, by definition, $\emptyset$ is a subset of every set.

If A is any set, then $\emptyset \subseteq A$.

Figure 1 shows a set A that is a subset of set B. The rectangle in the drawing represents the universal set U. Such diagrams are called **Venn diagrams.**



$A \subseteq B$

Figure 1

NOW TRY EXERCISES 55, 59, AND 63. ◀

Two sets A and B are equal whenever $A \subseteq B$ and $B \subseteq A$. In other words, $A = B$ if the two sets contain exactly the same elements. For example,

$$\{1, 2, 3\} = \{3, 1, 2\},$$

since both sets contain exactly the same elements. However,

$$\{1, 2, 3\} \neq \{0, 1, 2, 3\},$$

since the set $\{0, 1, 2, 3\}$ contains the element 0, which is not an element of $\{1, 2, 3\}$.

NOW TRY EXERCISES 35 AND 37.

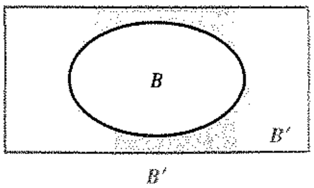


Figure 2

Operations on Sets Given a set A and a universal set U, the set of all elements of U that do not belong to set A is called the **complement** of set A. For example, if set A is the set of all the female students in your class, and U is the set of all students in the class, then the complement of A would be the set of all the male students in the class. The complement of set A is written A' (read "**A-prime**"). The Venn diagram in Figure 2 shows a set B. Its complement, B', is in color.

▶ EXAMPLE 2 FINDING THE COMPLEMENT OF A SET

Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5, 7\}$, and $B = \{3, 4, 6\}$. Find each set.

(a) A' **(b)** B' **(c)** $\emptyset'$ **(d)** U'

Solution

(a) Set A' contains the elements of U that are not in A: $A' = \{2, 4, 6\}$.

(b) $B' = \{1, 2, 5, 7\}$ **(c)** $\emptyset' = U$ **(d)** $U' = \emptyset$

NOW TRY EXERCISE 79.

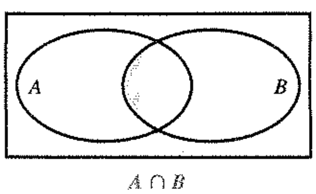


Figure 3

Given two sets A and B, the set of all elements belonging both to set A ***and*** to set B is called the **intersection** of the two sets, written $A \cap B$. For example, if $A = \{1, 2, 4, 5, 7\}$ and $B = \{2, 4, 5, 7, 9, 11\}$, then

$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}.$$

The Venn diagram in Figure 3 shows two sets A and B; their intersection, $A \cap B$, is in color.

▶ EXAMPLE 3 FINDING THE INTERSECTION OF TWO SETS

Find each of the following.

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\}$ **(b)** $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$

Solution

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\} = \{15, 25\}$
The elements 15 and 25 are the only ones belonging to both sets.

(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\} = \{2, 3, 4\}$

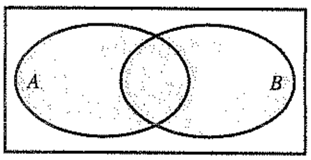
NOW TRY EXERCISES 41 AND 71.

Two sets that have no elements in common are called **disjoint sets.** For example, there are no elements common to both {50, 51, 54} and {52, 53, 55, 56}, so these two sets are disjoint, and $\{50, 51, 54\} \cap \{52, 53, 55, 56\} = \emptyset$.

DISJOINT SETS

If A and B are any two disjoint sets, then $A \cap B = \emptyset$.

NOW TRY EXERCISE 75.



$A \cup B$

Figure 4

The set of all elements belonging to set A ***or*** to set B (or to both) is called the **union** of the two sets, written $A \cup B$. For example,

$$\{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}.$$

The Venn diagram in Figure 4 shows two sets A and B; their union, $A \cup B$, is in color.

EXAMPLE 4 FINDING THE UNION OF TWO SETS

Find each of the following.

(a) $\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\}$ **(b)** $\{1, 3, 5, 7\} \cup \{2, 4, 6\}$

Solution

(a) Begin by listing the elements of the first set, {1, 2, 5, 9, 14}. Then include any elements from the second set that are not already listed. Doing this gives

$$\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\} = \{1, 2, 3, 4, 5, 8, 9, 14\}.$$

(b) $\{1, 3, 5, 7\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 5, 6, 7\}$

NOW TRY EXERCISES 43 AND 73.

The **set operations** summarized below are similar to operations on numbers, such as addition, subtraction, multiplication, or division.

SET OPERATIONS

For all sets A and B, with universal set U:

The **complement** of set A is the set A' of all elements in the universal set that do not belong to set A.

$$A' = \{x \mid x \in U, \quad x \notin A\}$$

The **intersection** of sets A and B, written $A \cap B$, is made up of all the elements belonging to both set A and set B.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

The **union** of sets A and B, written $A \cup B$, is made up of all the elements belonging to set A or to set B.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

R.1 Exercises

Use set notation, and list all the elements of each set. See Example 1.

1. $\{12, 13, 14, \ldots, 20\}$

2. $\{8, 9, 10, \ldots, 17\}$

3. $\left\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{32}\right\}$

4. $\{3, 9, 27, \ldots, 729\}$

5. $\{17, 22, 27, \ldots, 47\}$

6. $\{74, 68, 62, \ldots, 38\}$

7. {all natural numbers greater than 7 and less than 15}

8. {all natural numbers not greater than 4}

Identify the sets in Exercises 9–16 as finite *or* infinite.

9. $\{4, 5, 6, \ldots, 15\}$

10. $\{4, 5, 6, \ldots\}$

11. $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\right\}$

12. $\{0, 1, 2, 3, 4, 5, \ldots, 75\}$

13. $\{x \mid x$ is a natural number greater than $5\}$

14. $\{x \mid x$ is a person alive now$\}$

15. $\{x \mid x$ is a fraction between 0 and $1\}$

16. $\{x \mid x$ is an even natural number$\}$

Complete the blanks with either $\in$ *or* $\notin$ *so that the resulting statement is true.*

17. 6 ____ $\{3, 4, 5, 6\}$

18. 9 ____ $\{3, 2, 5, 9, 8\}$

19. -4 ____ $\{4, 6, 8, 10\}$

20. -12 ____ $\{3, 5, 12, 14\}$

21. 0 ____ $\{2, 0, 3, 4\}$

22. 0 ____ $\{5, 6, 7, 8, 10\}$

23. $\{3\}$ ____ $\{2, 3, 4, 5\}$

24. $\{5\}$ ____ $\{3, 4, 5, 6, 7\}$

25. $\{0\}$ ____ $\{0, 1, 2, 5\}$

26. $\{2\}$ ____ $\{2, 4, 6, 8\}$

27. 0 ____ $\emptyset$

28. $\emptyset$ ____ $\emptyset$

Tell whether each statement is true *or* false.

29. $3 \in \{2, 5, 6, 8\}$

30. $6 \in \{-2, 5, 8, 9\}$

31. $1 \in \{3, 4, 5, 11, 1\}$

32. $12 \in \{18, 17, 15, 13, 12\}$

33. $9 \notin \{2, 1, 5, 8\}$

34. $3 \notin \{7, 6, 5, 4\}$

35. $\{2, 5, 8, 9\} = \{2, 5, 9, 8\}$

36. $\{3, 0, 9, 6, 2\} = \{2, 9, 0, 3, 6\}$

37. $\{5, 8, 9\} = \{5, 8, 9, 0\}$

38. $\{3, 7, 12, 14\} = \{3, 7, 12, 14, 0\}$

39. $\{x \mid x$ is a natural number less than $3\} = \{1, 2\}$

40. $\{x \mid x$ is a natural number greater than $10\} = \{11, 12, 13, \ldots\}$

41. $\{5, 7, 9, 19\} \cap \{7, 9, 11, 15\} = \{7, 9\}$

42. $\{8, 11, 15\} \cap \{8, 11, 19, 20\} = \{8, 11\}$

43. $\{2, 1, 7\} \cup \{1, 5, 9\} = \{1\}$

44. $\{6, 12, 14, 16\} \cup \{6, 14, 19\} = \{6, 14\}$

45. $\{3, 2, 5, 9\} \cap \{2, 7, 8, 10\} = \{2\}$

46. $\{8, 9, 6\} \cup \{9, 8, 6\} = \{8, 9\}$

47. $\{3, 5, 9, 10\} \cap \emptyset = \{3, 5, 9, 10\}$

48. $\{3, 5, 9, 10\} \cup \emptyset = \{3, 5, 9, 10\}$

49. $\{1, 2, 4\} \cup \{1, 2, 4\} = \{1, 2, 4\}$

50. $\{1, 2, 4\} \cap \{1, 2, 4\} = \emptyset$

51. $\emptyset \cup \emptyset = \emptyset$

52. $\emptyset \cap \emptyset = \emptyset$

Let $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{2, 4, 8, 10\}$, $C = \{4, 10, 12\}$, $D = \{2, 10\}$, *and* $U = \{2, 4, 6, 8, 10, 12, 14\}$.

Tell whether each statement is true *or* false.

53. $A \subseteq U$ **54.** $C \subseteq U$ **55.** $D \subseteq B$ **56.** $D \subseteq A$
57. $A \subseteq B$ **58.** $B \subseteq C$ **59.** $\emptyset \subseteq A$ **60.** $\emptyset \subseteq \emptyset$
61. $\{4, 8, 10\} \subseteq B$ **62.** $\{0, 2\} \subseteq D$ **63.** $B \subseteq D$ **64.** $A \not\subseteq C$

Insert $\subseteq$ *or* $\not\subseteq$ *in each blank to make the resulting statement true.*

65. $\{2, 4, 6\}$ ____ $\{3, 2, 5, 4, 6\}$ **66.** $\{1, 5\}$ ____ $\{0, -1, 2, 3, 1, 5\}$
67. $\{0, 1, 2\}$ ____ $\{1, 2, 3, 4, 5\}$ **68.** $\{5, 6, 7, 8\}$ ____ $\{1, 2, 3, 4, 5, 6, 7\}$
69. $\emptyset$ ____ $\{1, 4, 6, 8\}$ **70.** $\emptyset$ ____ $\emptyset$

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$, $M = \{0, 2, 4, 6, 8\}$, $N = \{1, 3, 5, 7, 9, 11, 13\}$, $Q = \{0, 2, 4, 6, 8, 10, 12\}$, *and* $R = \{0, 1, 2, 3, 4\}$.

Use these sets to find each of the following. Identify any disjoint sets. See Examples 2–4.

71. $M \cap R$ **72.** $M \cup R$ **73.** $M \cup N$ **74.** $M \cap U$
75. $M \cap N$ **76.** $M \cup Q$ **77.** $N \cup R$ **78.** $U \cap N$
79. N' **80.** Q' **81.** $M' \cap Q$ **82.** $Q \cap R'$
83. $\emptyset \cap R$ **84.** $\emptyset \cap Q$ **85.** $N \cup \emptyset$ **86.** $R \cup \emptyset$
87. $(M \cap N) \cup R$ **88.** $(N \cup R) \cap M$ **89.** $(Q \cap M) \cup R$ **90.** $(R \cup N) \cap M'$
91. $(M' \cup Q) \cap R$ **92.** $Q \cap (M \cup N)$ **93.** $Q' \cap (N' \cap U)$ **94.** $(U \cap \emptyset') \cup R$

Let $U = \{$all students in this school$\}$, $M = \{$all students taking this course$\}$, $N = \{$all students taking calculus$\}$, *and* $P = \{$all students taking history$\}$.

Describe each set in words.

95. M' **96.** $M \cup N$ **97.** $N \cap P$
98. $N' \cap P'$ **99.** $M \cup P$ **100.** $P' \cup M'$

R.2 Real Numbers and Their Properties

Sets of Numbers and the Number Line ▪ Exponents ▪ Order of Operations ▪ Properties of Real Numbers ▪ Order on the Number Line ▪ Absolute Value

Sets of Numbers and the Number Line

When people first counted they used only the **natural numbers,** written in set notation as

$$\{1, 2, 3, 4, \ldots\}. \quad \text{Natural numbers (Section R.1)}$$

Including 0 with the set of natural numbers gives the set of **whole numbers,**

$$\{0, 1, 2, 3, 4, \ldots\}. \quad \text{Whole numbers}$$

Including the negatives of the natural numbers with the set of whole numbers gives the set of **integers,**

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}. \quad \text{Integers}$$