Let $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{2, 4, 8, 10\}$, $C = \{4, 10, 12\}$, $D = \{2, 10\}$, *and* $U = \{2, 4, 6, 8, 10, 12, 14\}$.

Tell whether each statement is true *or* false.

53. $A \subseteq U$ **54.** $C \subseteq U$ **55.** $D \subseteq B$ **56.** $D \subseteq A$

57. $A \subseteq B$ **58.** $B \subseteq C$ **59.** $\emptyset \subseteq A$ **60.** $\emptyset \subseteq \emptyset$

61. $\{4, 8, 10\} \subseteq B$ **62.** $\{0, 2\} \subseteq D$ **63.** $B \subseteq D$ **64.** $A \not\subseteq C$

Insert $\subseteq$ *or* $\not\subseteq$ *in each blank to make the resulting statement true.*

65. $\{2, 4, 6\}$ ____ $\{3, 2, 5, 4, 6\}$ **66.** $\{1, 5\}$ ____ $\{0, -1, 2, 3, 1, 5\}$

67. $\{0, 1, 2\}$ ____ $\{1, 2, 3, 4, 5\}$ **68.** $\{5, 6, 7, 8\}$ ____ $\{1, 2, 3, 4, 5, 6, 7\}$

69. $\emptyset$ ____ $\{1, 4, 6, 8\}$ **70.** $\emptyset$ ____ $\emptyset$

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$, $M = \{0, 2, 4, 6, 8\}$, $N = \{1, 3, 5, 7, 9, 11, 13\}$, $Q = \{0, 2, 4, 6, 8, 10, 12\}$, *and* $R = \{0, 1, 2, 3, 4\}$.

Use these sets to find each of the following. Identify any disjoint sets. See Examples 2–4.

71. $M \cap R$ **72.** $M \cup R$ **73.** $M \cup N$ **74.** $M \cap U$

75. $M \cap N$ **76.** $M \cup Q$ **77.** $N \cup R$ **78.** $U \cap N$

79. N' **80.** Q' **81.** $M' \cap Q$ **82.** $Q \cap R'$

83. $\emptyset \cap R$ **84.** $\emptyset \cap Q$ **85.** $N \cup \emptyset$ **86.** $R \cup \emptyset$

87. $(M \cap N) \cup R$ **88.** $(N \cup R) \cap M$ **89.** $(Q \cap M) \cup R$ **90.** $(R \cup N) \cap M'$

91. $(M' \cup Q) \cap R$ **92.** $Q \cap (M \cup N)$ **93.** $Q' \cap (N' \cap U)$ **94.** $(U \cap \emptyset') \cup R$

Let $U = \{\text{all students in this school}\}$, $M = \{\text{all students taking this course}\}$, $N = \{\text{all students taking calculus}\}$, *and* $P = \{\text{all students taking history}\}$.

Describe each set in words.

95. M' **96.** $M \cup N$ **97.** $N \cap P$

98. $N' \cap P'$ **99.** $M \cup P$ **100.** $P' \cup M'$

R.2 Real Numbers and Their Properties

Sets of Numbers and the Number Line ▪ Exponents ▪ Order of Operations ▪ Properties of Real Numbers ▪ Order on the Number Line ▪ Absolute Value

Sets of Numbers and the Number Line When people first counted they used only the **natural numbers,** written in set notation as

$$\{1, 2, 3, 4, \ldots\}.$$ Natural numbers (Section R.1)

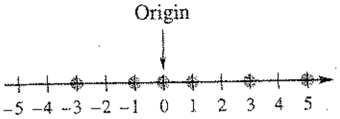
Including 0 with the set of natural numbers gives the set of **whole numbers,**

$$\{0, 1, 2, 3, 4, \ldots\}.$$ Whole numbers

Including the negatives of the natural numbers with the set of whole numbers gives the set of **integers,**

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$ Integers

Integers can be shown pictorially—that is, **graphed**—on a **number line.** See Figure 5. Every number corresponds to one and only one point on the number line, and each point corresponds to one and only one number. This correspondence is called a **coordinate system.** The number associated with a given point is called the **coordinate** of the point.



Graph of the Set $\{-3, -1, 0, 1, 3, 5\}$

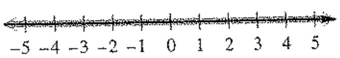
Figure 5

The result of dividing two integers (with a nonzero divisor) is called a *rational number,* or *fraction.* A **rational number** is an element of the set

$$\left\{\frac{p}{q}\,\middle|\, p \text{ and } q \text{ are integers and } q \neq 0\right\}. \quad \text{Rational numbers}$$

Rational numbers include the natural numbers, whole numbers, and integers. For example, the integer -3 is a rational number because it can be written as $\frac{-3}{1}$. Numbers that can be written as repeating or terminating decimals are also rational numbers. For example, $.\overline{6} = .66666\ldots$ represents a rational number that can be expressed as the fraction $\frac{2}{3}$.

The set of all numbers that correspond to points on a number line is called the **real numbers,** shown in Figure 6. Real numbers can be represented by decimals. Since every fraction has a decimal form—for example, $\frac{1}{4} = .25$—real numbers include rational numbers.



Graph of the Set of Real Numbers

Figure 6

Some real numbers cannot be represented by quotients of integers. These numbers are called **irrational numbers.** The set of irrational numbers includes $\sqrt{3}$ and $\sqrt{5}$, but not $\sqrt{1}, \sqrt{4}, \sqrt{9}, \ldots,$ which equal $1, 2, 3, \ldots,$ and hence are rational numbers. Another irrational number is π, which is approximately equal to 3.14159. The numbers in the set $\left\{-\frac{2}{3}, 0, \sqrt{2}, \sqrt{5}, \pi, 4\right\}$ can be located on a number line, as shown in Figure 7. (Only $\sqrt{2}$, $\sqrt{5}$, and π are irrational here. The others are rational.) Since $\sqrt{2}$ is approximately equal to 1.41, it is located between 1 and 2, slightly closer to 1.

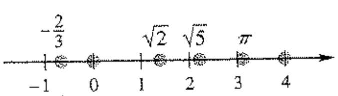
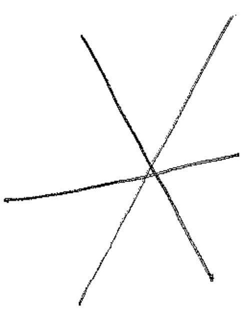


Figure 7

The sets of numbers discussed so far are summarized as follows.



SETS OF NUMBERS

Set	Description
Natural Numbers	$\{1, 2, 3, 4, \ldots\}$
Whole Numbers	$\{0, 1, 2, 3, 4, \ldots\}$
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational Numbers	$\{\frac{p}{q} \mid p$ and q are integers and $q \neq 0\}$
Irrational Numbers	$\{x \mid x$ is real but not rational$\}$
Real Numbers	$\{x \mid x$ corresponds to a point on a number line$\}$

▶ EXAMPLE 1 IDENTIFYING ELEMENTS OF SUBSETS OF THE REAL NUMBERS

Let set $A = \left\{-8, -6, -\frac{12}{4}, -\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}, 1, \sqrt{2}, \sqrt{5}, 6\right\}$. List the elements from set A that belong to each set.

(a) natural numbers **(b)** whole numbers **(c)** integers

(d) rational numbers **(e)** irrational numbers **(f)** real numbers

Solution

(a) The natural numbers in set A are 1 and 6.

(b) The whole numbers are 0, 1, and 6.

(c) The integers are $-8, -6, -\frac{12}{4}$ (or -3), 0, 1, and 6.

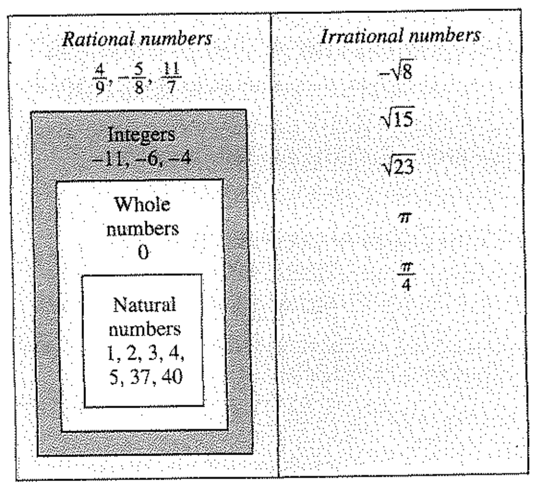
(d) The rational numbers are $-8, -6, -\frac{12}{4}$ (or -3), $-\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}$, 1, and 6.

(e) The irrational numbers are $\sqrt{2}$ and $\sqrt{5}$.

(f) All elements of A are real numbers.

NOW TRY EXERCISES 1, 11, AND 13.

The relationships among the subsets of the real numbers are shown in Figure 8.



The Real Numbers

Figure 8

Exponents

The product $2 \cdot 2 \cdot 2$ can be written as 2^3, where the 3 shows that three factors of 2 appear in the product. The notation a^n is defined as follows.

EXPONENTIAL NOTATION

If n is any positive integer and a is any real number, then the nth power of a is

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors of } a}.$$

That is, a^n means the product of n factors of a. The integer n is the **exponent,** a is the **base,** and a^n is a **power** or an **exponential expression** (or simply an **exponential**). Read a^n as **"a to the nth power,"** or just **"a to the nth."**

▶ EXAMPLE 2 EVALUATING EXPONENTIAL EXPRESSIONS

Evaluate each exponential expression, and identify the base and the exponent.

(a) 4^3 **(b)** $(-6)^2$ **(c)** -6^2 **(d)** $4 \cdot 3^2$ **(e)** $(4 \cdot 3)^2$

Solution

(a) $4^3 = \underbrace{4 \cdot 4 \cdot 4}_{\text{3 factors of 4}} = 64$ The base is 4 and the exponent is 3.

(b) $(-6)^2 = (-6)(-6) = 36$
The base is -6 and the exponent is 2.

(c) $-6^2 = -(6 \cdot 6) = -36$
The base is 6 and the exponent is 2.

(d) $4 \cdot 3^2 = 4 \cdot 3 \cdot 3 = 36$ The base is 3 and the exponent is 2.
$3^2 = 3 \cdot 3$, NOT $3 \cdot 2$.

(e) $(4 \cdot 3)^2 = 12^2 = 144$
The base is $4 \cdot 3$ or 12 and the exponent is 2.

NOW TRY EXERCISES 15, 17, 19, AND 21.◀

▶ **Caution** Notice in Examples 2(d) and (e) that

$$4 \cdot 3^2 \neq (4 \cdot 3)^2.$$

Order of Operations When a problem involves more than one operation symbol, we use the following order of operations.

ORDER OF OPERATIONS

If grouping symbols such as parentheses, square brackets, or fraction bars are present:

Step 1 Work separately above and below each **fraction bar.**

Step 2 Use the rules below within each set of **parentheses** or **square brackets.** Start with the innermost set and work outward.

If no grouping symbols are present:

Step 1 Simplify all **powers** and **roots,** *working from left to right.*

Step 2 Do any **multiplications** or **divisions** in order, *working from left to right.*

Step 3 Do any **negations, additions,** or **subtractions** in order, *working from left to right.*

▶ EXAMPLE 3 USING ORDER OF OPERATIONS

Evaluate each expression.

(a) $6 \div 3 + 2^3 \cdot 5$ **(b)** $(8 + 6) \div 7 \cdot 3 - 6$

(c) $\dfrac{4 + 3^2}{6 - 5 \cdot 3}$ **(d)** $\dfrac{-(-3)^3 + (-5)}{2(-8) - 5(3)}$

Solution

(a)
$$
\begin{aligned}
6 \div 3 + 2^3 \cdot 5 &= 6 \div 3 + 8 \cdot 5 && \text{Evaluate the exponential.}\\
&= 2 + 8 \cdot 5 && \text{Divide.}\\
&= 2 + 40 && \text{Multiply.}\\
&= 42 && \text{Add.}
\end{aligned}
$$

Multiply or divide in order from left to right.

(b)
$$
\begin{aligned}
(8 + 6) \div 7 \cdot 3 - 6 &= 14 \div 7 \cdot 3 - 6 && \text{Work within the parentheses.}\\
&= 2 \cdot 3 - 6 && \text{Divide.}\\
&= 6 - 6 && \text{Multiply.}\\
&= 0 && \text{Subtract.}
\end{aligned}
$$

(c)
$$
\begin{aligned}
\frac{4 + 3^2}{6 - 5 \cdot 3} &= \frac{4 + 9}{6 - 15} && \text{Evaluate the exponential and multiply.}\\
&= \frac{13}{-9}, \text{ or } -\frac{13}{9} && \text{Add and subtract; } \tfrac{a}{-b} = -\tfrac{a}{b}.
\end{aligned}
$$

(d)
$$
\begin{aligned}
\frac{-(-3)^3 + (-5)}{2(-8) - 5(3)} &= \frac{-(-27) + (-5)}{2(-8) - 5(3)} && \text{Evaluate the exponential.}\\
&= \frac{27 + (-5)}{-16 - 15} && \text{Multiply.}\\
&= \frac{22}{-31}, \text{ or } -\frac{22}{31} && \text{Add and subtract; } \tfrac{a}{-b} = -\tfrac{a}{b}.
\end{aligned}
$$

NOW TRY EXERCISES 23, 25, AND 31. ◀

▶ EXAMPLE 4 USING ORDER OF OPERATIONS

Evaluate each expression if $x = -2$, $y = 5$, and $z = -3$.

(a) $-4x^2 - 7y + 4z$ **(b)** $\dfrac{2(x - 5)^2 + 4y}{z + 4}$

Solution

(a)
$$
\begin{aligned}
-4x^2 - 7y + 4z &= -4(-2)^2 - 7(5) + 4(-3) && \text{Substitute: } x = -2,\ y = 5, \text{ and } z = -3.\\
&= -4(4) - 7(5) + 4(-3) && \text{Evaluate the exponential.}\\
&= -16 - 35 - 12 && \text{Multiply.}\\
&= -63 && \text{Subtract.}
\end{aligned}
$$

Use parentheses around substituted values to avoid errors.

(b) $\dfrac{2(x-5)^2+4y}{z+4} = \dfrac{2(-2-5)^2+4(5)}{-3+4}$ Substitute: $x=-2$, $y=5$, and $z=-3$.

$= \dfrac{2(-7)^2+20}{1}$ Work inside parentheses; multiply; add.

$= 2(49)+20$ Evaluate the exponential.

$= 98+20$ Multiply.

$= 118$ Add.

NOW TRY EXERCISES 33 AND 39.

Properties of Real Numbers The following basic properties can be generalized to apply to expressions with variables.

PROPERTIES OF REAL NUMBERS

For all real numbers a, b, and c:

Property	Description
Closure Properties $a+b$ is a real number. ab is a real number.	The sum or product of two real numbers is a real number.
Commutative Properties $a+b=b+a$ $ab=ba$	The sum or product of two real numbers is the same regardless of their order.
Associative Properties $(a+b)+c=a+(b+c)$ $(ab)c=a(bc)$	The sum or product of three real numbers is the same no matter which two are added or multiplied first.
Identity Properties There exists a unique real number 0 such that $a+0=a$ and $0+a=a$. There exists a unique real number 1 such that $a \cdot 1=a$ and $1 \cdot a=a$.	The sum of a real number and 0 is that real number, and the product of a real number and 1 is that real number.
Inverse Properties There exists a unique real number $-a$ such that $a+(-a)=0$ and $-a+a=0$. If $a \neq 0$, there exists a unique real number $\frac{1}{a}$ such that $a \cdot \frac{1}{a}=1$ and $\frac{1}{a} \cdot a=1$.	The sum of any real number and its negative is 0, and the product of any nonzero real number and its reciprocal is 1.
Distributive Properties $a(b+c)=ab+ac$ $a(b-c)=ab-ac$	The product of a real number and the sum (or difference) of two real numbers equals the sum (or difference) of the products of the first number and each of the other numbers.

▶ **Caution** Notice with the commutative properties that the ***order*** changes from one side of the equality symbol to the other; with the associative properties the order does not change, but the ***grouping*** does.

Commutative Properties	Associative Properties
$(x + 4) + 9 = (4 + x) + 9$	$(x + 4) + 9 = x + (4 + 9)$
$7 \cdot (5 \cdot 2) = (5 \cdot 2) \cdot 7$	$7 \cdot (5 \cdot 2) = (7 \cdot 5) \cdot 2$

▶ EXAMPLE 5 USING THE COMMUTATIVE AND ASSOCIATIVE PROPERTIES TO SIMPLIFY EXPRESSIONS

Simplify each expression.

(a) $6 + (9 + x)$ **(b)** $\frac{5}{8}(16y)$ **(c)** $-10p\left(\frac{6}{5}\right)$

Solution

(a) $6 + (9 + x) = (6 + 9) + x = 15 + x$ Associative property

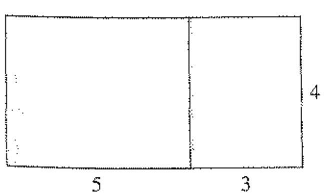
(b) $\frac{5}{8}(16y) = \left(\frac{5}{8} \cdot 16\right)y = 10y$ Associative property

(c) $-10p\left(\frac{6}{5}\right) = \frac{6}{5}(-10p)$ Commutative property

$= \left[\frac{6}{5}(-10)\right]p$ Associative property

$= -12p$ Multiply.

NOW TRY EXERCISES 61 AND 63. ◀



Geometric Model of the Distributive Property

Figure 9

Figure 9 helps to explain the distributive property. The area of the entire region shown can be found in two ways. We can multiply the length of the base of the entire region, $5 + 3 = 8$, by the width of the region.

$$4(5 + 3) = 4(8) = 32$$

Or, we can add the areas of the smaller rectangles, $4(5) = 20$ and $4(3) = 12$.

$$4(5) + 4(3) = 20 + 12 = 32$$

The result is the same. This means that

$$4(5 + 3) = 4(5) + 4(3).$$

▶ **Note** The distributive property is a key property of real numbers because it is used to change products to sums and sums to products.

▶ EXAMPLE 6 USING THE DISTRIBUTIVE PROPERTY

Rewrite each expression using the distributive property and simplify, if possible.

(a) $3(x + y)$ **(b)** $-(m - 4n)$

(c) $\frac{1}{3}\left(\frac{4}{5}m - \frac{3}{2}n - 27\right)$ **(d)** $7p + 21$

Solution

(a) $3(x + y) = 3x + 3y$

(b)
$$\begin{aligned} -(m - 4n) &= -1(m - 4n) \\ &= -1(m) + (-1)(-4n) \\ &= -m + 4n \end{aligned}$$

Be careful with the negative signs.

(c)
$$\begin{aligned} \frac{1}{3}\left(\frac{4}{5}m - \frac{3}{2}n - 27\right) &= \frac{1}{3}\left(\frac{4}{5}m\right) + \frac{1}{3}\left(-\frac{3}{2}n\right) + \frac{1}{3}(-27) \\ &= \frac{4}{15}m - \frac{1}{2}n - 9 \end{aligned}$$

(d)
$$\begin{aligned} 7p + 21 &= 7p + 7 \cdot 3 \\ &= 7(p + 3) \end{aligned}$$

NOW TRY EXERCISES 57, 59, AND 65. ◀

Order on the Number Line If the real number a is to the left of the real number b on a number line, then

$$a \textit{ is less than } b, \quad \text{written} \quad a < b.$$

If a is to the right of b, then

$$a \textit{ is greater than } b, \quad \text{written} \quad a > b.$$

The inequality symbol must point toward the lesser number.

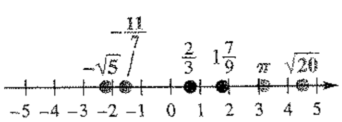


Figure 10

For example, in Figure 10, $-\sqrt{5}$ is to the left of $-\frac{11}{7}$ on the number line, so $-\sqrt{5} < -\frac{11}{7}$, and $\sqrt{20}$ is to the right of π, indicating $\sqrt{20} > \pi$.

Statements involving these symbols, as well as the symbols less than or equal to, $\leq$, and greater than or equal to, $\geq$, are called **inequalities.** The inequality $a < b < c$ says that b is ***between*** a and c since $a < b$ and $b < c$.

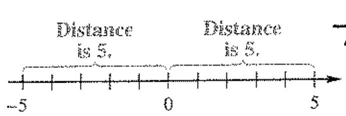


Figure 11

Absolute Value The distance on the number line from a number to 0 is called the **absolute value** of that number. The absolute value of the number a is written $|a|$. For example, the distance on the number line from 5 to 0 is 5, as is the distance from −5 to 0. (See Figure 11.) Therefore,

$$|5| = 5 \quad \text{and} \quad |-5| = 5.$$

▶ **Note** Since distance cannot be negative, ***the absolute value of a number is always positive or 0.***

The algebraic definition of absolute value follows.

ABSOLUTE VALUE

For all real numbers a,

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0. \end{cases}$$

That is, the absolute value of a positive number or 0 equals that number; the absolute value of a negative number equals its negative (or opposite).

▶ EXAMPLE 7 EVALUATING ABSOLUTE VALUES

Evaluate each expression.

(a) $\left|-\frac{5}{8}\right|$ **(b)** $-|8|$ **(c)** $-|-2|$ **(d)** $|2x|$, if $x = \pi$

Solution

(a) $\left|-\frac{5}{8}\right| = \frac{5}{8}$

(b) $-|8| = -(8) = -8$

(c) $-|-2| = -(2) = -2$

(d) $|2\pi| = 2\pi$

NOW TRY EXERCISES 77 AND 79. ◀

Absolute value is useful in applications where only the *size* (or magnitude), not the *sign*, of the difference between two numbers is important.

▶ EXAMPLE 8 MEASURING BLOOD PRESSURE DIFFERENCE

Systolic blood pressure is the maximum pressure produced by each heartbeat. Both low blood pressure and high blood pressure may be cause for medical concern. Therefore, health care professionals are interested in a patient's "pressure difference from normal," or P_d. If 120 is considered a normal systolic pressure, $P_d = |P - 120|$, where P is the patient's recorded systolic pressure. Find P_d for a patient with a systolic pressure, P, of 113.

Solution

$$\begin{aligned} P_d &= |P - 120| & \\ &= |113 - 120| & \text{Let } P = 113. \\ &= |-7| & \text{Subtract.} \\ &= 7 & \text{Definition of absolute value} \end{aligned}$$

NOW TRY EXERCISE 97. ◀

The definition of absolute value can be used to prove the following.

PROPERTIES OF ABSOLUTE VALUE

For all real numbers a and b:

Property	Description
1. $\lvert a \rvert \geq 0$	The absolute value of a real number is positive or 0.
2. $\lvert -a \rvert = \lvert a \rvert$	The absolute values of a real number and its opposite are equal.
3. $\lvert a \rvert \cdot \lvert b \rvert = \lvert ab \rvert$	The product of the absolute values of two real numbers equals the absolute value of their product.
4. $\dfrac{\lvert a \rvert}{\lvert b \rvert} = \left\lvert \dfrac{a}{b} \right\rvert \quad (b \neq 0)$	The quotient of the absolute values of two real numbers equals the absolute value of their quotient.
5. $\lvert a + b \rvert \leq \lvert a \rvert + \lvert b \rvert$ (the triangle inequality)	The absolute value of the sum of two real numbers is less than or equal to the sum of their absolute values.

▼ LOOKING AHEAD TO CALCULUS

One of the most important definitions in calculus, that of the **limit,** uses absolute value.

Suppose that a function f is defined at every number in an open interval I containing a, except perhaps at a itself. Then the limit of $f(x)$ as x approaches a is L, written

$$\lim_{x \to a} f(x) = L,$$

if for every $\epsilon > 0$ there exists a $\delta > 0$ such that $\lvert f(x) - L \rvert < \epsilon$ whenever $0 < \lvert x - a \rvert < \delta$.

To illustrate these properties, see the following.

$$\lvert -15 \rvert = 15 \geq 0 \qquad \text{Property 1}$$

$$\lvert -10 \rvert = 10 \text{ and } \lvert 10 \rvert = 10, \text{ so } \lvert -10 \rvert = \lvert 10 \rvert. \qquad \text{Property 2}$$

$$\lvert 5x \rvert = \lvert 5 \rvert \cdot \lvert x \rvert = 5\lvert x \rvert \text{ since 5 is positive.} \qquad \text{Property 3}$$

$$\left\lvert \frac{2}{y} \right\rvert = \frac{\lvert 2 \rvert}{\lvert y \rvert} = \frac{2}{\lvert y \rvert}, \quad y \neq 0 \qquad \text{Property 4}$$

To illustrate the triangle inequality, we let $a = 3$ and $b = -7$.

$$\lvert a + b \rvert = \lvert 3 + (-7) \rvert = \lvert -4 \rvert = 4$$

$$\lvert a \rvert + \lvert b \rvert = \lvert 3 \rvert + \lvert -7 \rvert = 3 + 7 = 10$$

Thus, $\lvert a + b \rvert \leq \lvert a \rvert + \lvert b \rvert$. Property 5

NOW TRY EXERCISES 89, 91, AND 93.

▶ EXAMPLE 9 EVALUATING ABSOLUTE VALUE EXPRESSIONS

Let $x = -6$ and $y = 10$. Evaluate each expression.

(a) $\lvert 2x - 3y \rvert$ **(b)** $\dfrac{2\lvert x \rvert - \lvert 3y \rvert}{\lvert xy \rvert}$

Solution

(a)
$$\begin{aligned} \lvert 2x - 3y \rvert &= \lvert 2(-6) - 3(10) \rvert && \text{Substitute.} \\ &= \lvert -12 - 30 \rvert && \text{Work inside absolute value bars; multiply.} \\ &= \lvert -42 \rvert && \text{Subtract.} \\ &= 42 \end{aligned}$$

(b) $\frac{2|x| - |3y|}{|xy|} = \frac{2|-6| - |3(10)|}{|-6(10)|}$ Substitute.

$= \frac{2 \cdot 6 - |30|}{|-60|}$ $|-6| = 6$; multiply.

$= \frac{12 - 30}{60}$ Multiply; $|30| = 30$; $|-60| = 60$.

$= \frac{-18}{60}$ Subtract.

$= -\frac{3}{10}$ Lowest terms; $\frac{-a}{b} = -\frac{a}{b}$

NOW TRY EXERCISES 83 AND 85.

Absolute value is used to find the distance between two points on a number line.

DISTANCE BETWEEN POINTS ON A NUMBER LINE

If P and Q are points on the number line with coordinates a and b, respectively, then the distance $d(P, Q)$ between them is

$$d(P,Q) = |b - a| \quad \text{or} \quad d(P,Q) = |a - b|.$$

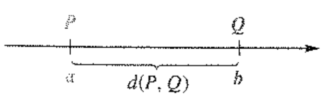


Figure 12

That is, the distance between two points on a number line is the absolute value of the difference between their coordinates in either order. See Figure 12. For example, the distance between -5 and 8 is given by

$$|8 - (-5)| = |8 + 5| = |13| = 13.$$

Alternatively,

$$|(-5) - 8| = |-13| = 13.$$

NOW TRY EXERCISE 103.

R.2 Exercises

1. *Concept Check* Match each number from Column I with the letter or letters of the sets of numbers from Column II to which the number belongs. There may be more than one choice, so give all choices.

I	II
(a) 0	A. Natural numbers
(b) 34	B. Whole numbers
(c) $-\frac{9}{4}$	C. Integers
(d) $\sqrt{36}$	D. Rational numbers
(e) $\sqrt{13}$	E. Irrational numbers
(f) 2.16	F. Real numbers

2. Explain why no answer in Exercise 1 can contain both D and E as choices.

Concept Check *Decide whether each statement is* true *or* false. *If it is false, tell why.*

3. Every integer is a whole number.
4. Every natural number is an integer.
5. Every irrational number is an integer.
6. Every integer is a rational number.
7. Every natural number is a whole number.
8. Some rational numbers are irrational.
9. Some rational numbers are whole numbers.
10. Some real numbers are integers.

Let set $B = \left\{-6, -\frac{12}{4}, -\frac{5}{8}, -\sqrt{3}, 0, \frac{1}{4}, 1, 2\pi, 3, \sqrt{12}\right\}$. *List all the elements of B that belong to each set. See Example 1.*

11. Natural numbers
12. Whole numbers
13. Integers
14. Rational numbers

Evaluate each expression. See Example 2.

15. -2^4
16. -3^5
17. $(-2)^4$
18. -2^6
19. $(-3)^5$
20. $(-2)^5$
21. $-2 \cdot 3^4$
22. $-4(-5)^3$

Evaluate each expression. See Example 3.

23. $-2 \cdot 5 + 12 \div 3$
24. $9 \cdot 3 - 16 \div 4$
25. $-4(9 - 8) + (-7)(2)^3$
26. $6(-5) - (-3)(2)^4$
27. $(4 - 2^3)\left(-2 + \sqrt{25}\right)$
28. $\left[-3^2 - (-2)\right]\left[\sqrt{16} - 2^3\right]$
29. $\left(-\frac{2}{9} - \frac{1}{4}\right) - \left[-\frac{5}{18} - \left(-\frac{1}{2}\right)\right]$
30. $\left[-\frac{5}{8} - \left(-\frac{2}{5}\right)\right] - \left(\frac{3}{2} - \frac{11}{10}\right)$
31. $\dfrac{-8 + (-4)(-6) \div 12}{4 - (-3)}$
32. $\dfrac{15 \div 5 \cdot 4 \div 6 - 8}{-6 - (-5) - 8 \div 2}$

Evaluate each expression if $p = -4$, $q = 8$, *and* $r = -10$. *See Example 4.*

33. $2p - 7q + r^2$
34. $-p^3 - 2q + r$
35. $\dfrac{q + r}{q + p}$
36. $\dfrac{3q}{3p - 2r}$
37. $\dfrac{3q}{r} - \dfrac{5}{p}$
38. $\dfrac{\frac{q}{4} - \frac{r}{5}}{\frac{p}{2} + \frac{q}{2}}$
39. $\dfrac{-(p + 2)^2 - 3r}{2 - q}$
40. $\dfrac{5q + 2(1 + p)^3}{r + 3}$

Passing Rating for NFL Quarterbacks *Use the formula*

$$\text{Passing Rating} \approx 85.68\left(\frac{C}{A}\right) + 4.31\left(\frac{Y}{A}\right) + 326.42\left(\frac{T}{A}\right) - 419.07\left(\frac{I}{A}\right),$$

where A = *number of passes attempted,* C = *number of passes completed,* Y = *total number of yards gained passing,* T = *number of touchdown passes, and* I = *number of interceptions, to approximate the passing rating for each NFL quarterback in Exercises 41–44. (The formula is exact to one decimal place in Exercises 41–43 and in Exercise 44 differs by only .1.) (Source:* www.NFL.com*)*

NFL Quarterback	A	C	Y	T	I
41. Brad Johnson	451	281	3049	22	6
42. Trent Green	470	287	3690	26	13
43. Drew Bledsoe	610	375	4359	24	15
44. Peyton Manning	591	392	4200	27	19

Blood Alcohol Concentration *The Blood Alcohol Concentration (BAC) of a person who has been drinking is given by the expression*

number of oz $\times$ % alcohol $\times$ .075 $\div$ body weight in lb $-$ hr of drinking $\times$ .015.

(*Source:* Lawlor, J., *Auto Math Handbook: Mathematical Calculations, Theory, and Formulas for Automotive Enthusiasts,* HP Books, 1991.)

45. Suppose a policeman stops a 190-lb man who, in 2 hr, has ingested four 12-oz beers (48 oz), each having a 3.2% alcohol content. Calculate the man's BAC to the nearest thousandth. Follow the order of operations.

46. Find the BAC to the nearest thousandth for a 135-lb woman who, in 3 hr, has drunk three 12-oz beers (36 oz), each having a 4.0% alcohol content.

47. Calculate the BACs in Exercises 45 and 46 if each person weighs 25 lb more and the rest of the variables stay the same. How does increased weight affect a person's BAC?

48. Predict how decreased weight would affect the BAC of each person in Exercises 45 and 46. Calculate the BACs if each person weighs 25 lb less and the rest of the variables stay the same.

Identify the property illustrated in each statement. Assume all variables represent real numbers. See Examples 5 and 6.

49. $6 \cdot 12 + 6 \cdot 15 = 6(12 + 15)$

50. $8(m + 4) = (m + 4) \cdot 8$

51. $(t - 6) \cdot \left(\frac{1}{t - 6}\right) = 1$, if $t - 6 \neq 0$

52. $\frac{2 + m}{2 - m} \cdot \frac{2 - m}{2 + m} = 1$, if $m \neq 2$ or -2

53. $(7.5 - y) + 0 = 7.5 - y$

54. $1 + \pi$ is a real number.

55. Is there a commutative property for subtraction? That is, in general, is $a - b$ equal to $b - a$? Support your answer with examples.

56. Is there an associative property for subtraction? That is, does $(a - b) - c$ equal $a - (b - c)$ in general? Support your answer with examples.

Use the distributive property to rewrite sums as products and products as sums. See Example 6.

57. $8p - 14p$

58. $15x - 10x$

59. $-4(z - y)$

60. $-3(m + n)$

Simplify each expression. See Examples 5 and 6.

61. $\frac{10}{11}(22z)$

62. $\left(\frac{3}{4}r\right)(-12)$

63. $(m + 5) + 6$

64. $8 + (a + 7)$

65. $\frac{3}{8}\left(\frac{16}{9}y + \frac{32}{27}z - \frac{40}{9}\right)$

66. $-\frac{1}{4}(20m + 8y - 32z)$

Concept Check *Use the distributive property to calculate each value mentally.*

67. $72 \cdot 17 + 28 \cdot 17$

68. $32 \cdot 80 + 32 \cdot 20$

69. $123\frac{5}{8} \cdot 1\frac{1}{2} - 23\frac{5}{8} \cdot 1\frac{1}{2}$

70. $17\frac{2}{5} \cdot 14\frac{3}{4} - 17\frac{2}{5} \cdot 4\frac{3}{4}$

Concept Check *Decide whether each statement is* true *or* false. *If false, correct the statement so it is true.*

71. $|6 - 8| = |6| - |8|$

72. $|(-3)^3| = -|3^3|$

73. $|-5| \cdot |6| = |-5 \cdot 6|$

74. $\frac{|-14|}{|2|} = \left|\frac{-14}{2}\right|$

75. $|a - b| = |a| - |b|$, if $b > a > 0$.

76. If a is negative, then $|a| = -a$.

Evaluate each expression. See Example 7.

77. $|-10|$

78. $|-15|$

79. $-\left|\frac{4}{7}\right|$

80. $-\left|\frac{7}{2}\right|$

Let $x = -4$ and $y = 2$. Evaluate each expression. See Example 9.

81. $|x - y|$

82. $|2x + 5y|$

83. $|3x + 4y|$

84. $|-5y + x|$

85. $\frac{2|y| - 3|x|}{|xy|}$

86. $\frac{4|x| + 4|y|}{|x|}$

87. $\frac{|-8y + x|}{-|x|}$

88. $\frac{|x| + 2|y|}{5 + x}$

Justify each statement by giving the correct property of absolute value from page 16. Assume all variables represent real numbers.

89. $|m| = |-m|$

90. $|-k| \geq 0$

91. $|9| \cdot |-6| = |-54|$

92. $|k - m| \leq |k| + |-m|$

93. $|12 + 11r| \geq 0$

94. $\left|\frac{-12}{5}\right| = \frac{|-12|}{|5|}$

Solve each problem.

95. *Golf Scores* In the 2007 Masters Golf Tournament, Zach Johnson won the final round with a score that was 3 under par, while the 2006 tournament winner, Phil Mickelson, finished with a final-round score that was 5 over par. Using -3 to represent 3 under par and $+5$ to represent 5 over par, find the difference between these scores (in either order) and take the absolute value of this difference. What does this final number represent? (*Source:* www.masters.org)

96. *Total Football Yardage* During his 16 yr in the NFL, Marcus Allen gained 12,243 yd rushing, 5411 yd receiving, and -6 yd returning fumbles. Find his total yardage (called *all-purpose yards*). Is this the same as the sum of the absolute values of the three categories? Why or why not? (*Source: The Sports Illustrated 2003 Sports Almanac,* 2003.)

97. *Blood Pressure Difference* Calculate the P_d value for a woman whose actual systolic pressure is 116 and whose normal value should be 125. (See Example 8.)

98. *Systolic Blood Pressure* If a patient's P_d value is 17 and the normal pressure for his gender and age should be 130, what are the two possible values for his systolic blood pressure? (See Example 8.)

Windchill *The windchill factor is a measure of the cooling effect that the wind has on a person's skin. It calculates the equivalent cooling temperature if there were no wind. The chart gives the windchill factor for various wind speeds and temperatures at which frostbite is a risk, and how quickly it may occur.*

Temperature (°F)

Wind speed (mph) / Calm	40	30	20	10	0	−10	−20	−30	−40
5	36	25	13	1	−11	−22	−34	−46	−57
10	34	21	9	−4	−16	−28	−41	−53	−66
15	32	19	6	−7	−19	−32	−45	−58	−71
20	30	17	4	−9	−22	−35	−48	−61	−74
25	29	16	3	−11	−24	−37	−51	−64	−78
30	28	15	1	−12	−26	−39	−53	−67	−80
35	28	14	0	−14	−27	−41	−55	−69	−82
40	27	13	−1	−15	−29	−43	−57	−71	−84

□ 30 minutes ▨ 10 minutes ▩ 5 minutes

Source: National Oceanic and Atmospheric Administration, National Weather Service.

If we are interested only in the magnitude of the difference between two of these entries, then we subtract the two entries and find the absolute value. Find the magnitude of the difference for each pair of windchill factors.

99. wind at 15 mph with a 30°F temperature and wind at 10 mph with a −10°F temperature

100. wind at 20 mph with a −20°F temperature and wind at 5 mph with a 30°F temperature

101. wind at 30 mph with a −30°F temperature and wind at 15 mph with a −20°F temperature

102. wind at 40 mph with a 40°F temperature and wind at 25 mph with a −30°F temperature

Find the given distances between points P, Q, R, and S on a number line, with coordinates −4, −1, 8, *and* 12, *respectively.*

103. $d(P, Q)$

104. $d(P, R)$

105. $d(Q, R)$

106. $d(Q, S)$

Concept Check *Determine what signs on values of x and y would make each statement true. Assume that x and y are not* 0. *(You should be able to work mentally.)*

107. $xy > 0$

108. $x^2y > 0$

109. $\frac{x}{y} < 0$

110. $\frac{y^2}{x} < 0$

111. $\frac{x^3}{y} > 0$

112. $-\frac{x}{y} > 0$