R.3 Polynomials

Rules for Exponents ▪ Polynomials ▪ Addition and Subtraction ▪ Multiplication ▪ Division

Rules for Exponents

Work with exponents is simplified by using rules for exponents. From **Section R.2,** the notation a^m (where m is a positive integer and a is a real number) means that a appears as a factor m times. In the same way, a^n (where n is a positive integer) means that a appears as a factor n times. In the product $a^m \cdot a^n$, the number a would appear $m + n$ times, so the **product rule** states that

$$a^m \cdot a^n = a^{m+n}.$$

▶ EXAMPLE 1 USING THE PRODUCT RULE

Find each product.

(a) $y^4 \cdot y^7$

(b) $(6z^5)(9z^3)(2z^2)$

Solution

(a) $y^4 \cdot y^7 = y^{4+7} = y^{11}$ Product rule; keep the base, add the exponents.

(b) $(6z^5)(9z^3)(2z^2) = (6 \cdot 9 \cdot 2) \cdot (z^5z^3z^2)$ Commutative and associative properties **(Section R.2)**

$= 108z^{5+3+2}$ Product rule

$= 108z^{10}$

NOW TRY EXERCISES 11 AND 17. ◀

The expression $(2^5)^3$ can be written as

$$(2^5)^3 = 2^5 \cdot 2^5 \cdot 2^5.$$

By a generalization of the product rule for exponents, this product is

$$(2^5)^3 = 2^{5+5+5} = 2^{15}.$$

The same exponent could have been obtained by multiplying 3 and 5. This example suggests the first of the **power rules** below. The others are found in a similar way. For positive integers m and n and all real numbers a and b,

1. $(a^m)^n = a^{mn}$ **2.** $(ab)^m = a^mb^m$ **3.** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $(b \neq 0)$.

▶ EXAMPLE 2 USING THE POWER RULES

Simplify. Assume all variables represent nonzero real numbers.

(a) $(5^3)^2$ **(b)** $(3^4x^2)^3$ **(c)** $\left(\frac{2^5}{b^4}\right)^3$ **(d)** $\left(\frac{-2m^6}{t^2z}\right)^5$

Solution

(a) $(5^3)^2 = 5^{3(2)} = 5^6$ Power rule 1

(b) $(3^4x^2)^3 = (3^4)^3(x^2)^3$ Power rule 2

$= 3^{4(3)}x^{2(3)}$ Power rule 1

$= 3^{12}x^6$

(c) $\left(\frac{2^5}{b^4}\right)^3 = \frac{(2^5)^3}{(b^4)^3}$ Power rule 3

$= \frac{2^{15}}{b^{12}}$ Power rule 1

(d) $\left(\frac{-2m^6}{t^2z}\right)^5 = \frac{(-2m^6)^5}{(t^2z)^5}$ Power rule 3

$= \frac{(-2)^5(m^6)^5}{(t^2)^5z^5}$ Power rule 2

$= \frac{-32m^{30}}{t^{10}z^5}$, or $-\frac{32m^{30}}{t^{10}z^5}$ Evaluate $(-2)^5$; power rule 1

NOW TRY EXERCISES 19, 25, AND 27. ◀

▶ **Caution** The expressions mn^2 and $(mn)^2$ are ***not*** equal. The second power rule can be used only with the second expression: $(mn)^2 = m^2n^2$.

These rules for exponents are summarized here.

RULES FOR EXPONENTS

For all positive integers m and n and all real numbers a and b:

Rule	Description
Product Rule $a^m \cdot a^n = a^{m+n}$	When multiplying powers of like bases, keep the base and add the exponents.
Power Rule 1 $(a^m)^n = a^{mn}$	To raise a power to a power, multiply exponents.
Power Rule 2 $(ab)^m = a^mb^m$	To raise a product to a power, raise each factor to that power.
Power Rule 3 $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0)$	To raise a quotient to a power, raise the numerator and the denominator to that power.

A zero exponent is defined as follows.

ZERO EXPONENT

For any nonzero real number a, $a^0 = 1$.

That is, any nonzero number with a zero exponent equals 1. We show why a^0 is defined this way in **Section R.6.** ***The symbol* 0^0 *is undefined.***

▶ EXAMPLE 3 USING THE DEFINITION OF a^0

Evaluate each power.

(a) 4^0 **(b)** $(-4)^0$ **(c)** -4^0 **(d)** $-(-4)^0$ **(e)** $(7r)^0$

Solution

(a) $4^0 = 1$ Base is 4. **(b)** $(-4)^0 = 1$ Base is −4.

(c) $-4^0 = -(4^0) = -1$ Base is 4. **(d)** $-(-4)^0 = -(1) = -1$ Base is −4.

(e) $(7r)^0 = 1,\ r \neq 0$ Base is $7r$.

NOW TRY EXERCISE 29. ◀

Polynomials An **algebraic expression** is the result of adding, subtracting, multiplying, dividing (except by 0), raising to powers, or taking roots on any combination of variables, such as x, y, m, a, and b, or constants, such as −2, 3, 15, and 64.

$$-2x^2 + 3x, \quad \frac{15y}{2y-3}, \quad \sqrt{m^3 - 64}, \quad (3a + b)^4 \quad \text{Algebraic expressions}$$

The simplest algebraic expressions, *polynomials,* are discussed in this section.

The product of a real number and one or more variables raised to powers is called a **term.** The real number is called the **numerical coefficient,** or just the **coefficient.** The coefficient in $-3m^4$ is −3, while the coefficient in $-p^2$ is −1. **Like terms** are terms with the same variables each raised to the same powers.

$$-13x^3, \quad 4x^3, \quad -x^3 \quad \text{Like terms}$$

$$6y, \quad 6y^2, \quad 4y^3 \quad \text{Unlike terms}$$

A **polynomial** is defined as a term or a finite sum of terms, with only positive or zero integer exponents permitted on the variables. If the terms of a polynomial contain only the variable x, then the polynomial is called a **polynomial in x.** (Polynomials in other variables are defined similarly.)

$$5x^3 - 8x^2 + 7x - 4, \quad 9p^5 - 3, \quad 8r^2, \quad 6 \quad \text{Polynomials}$$

The terms of a polynomial cannot have variables in a denominator.

$$9x^2 - 4x + \frac{6}{x} \quad \text{Not a polynomial}$$

The **degree of a term** with one variable is the exponent on the variable. For example, the degree of $2x^3$ is 3, the degree of $-x^4$ is 4, and the degree of $17x$ (that is, $17x^1$) is 1. The greatest degree of any term in a polynomial is called the **degree of the polynomial.** For example,

$$4x^3 - 2x^2 - 3x + 7 \text{ has degree } 3,$$

because the greatest degree of any term is 3 (the degree of $4x^3$). A nonzero constant such as −6, which can be written as $-6x^0$, has degree 0. (The polynomial 0 has no degree.)

A polynomial can have more than one variable. A term containing more than one variable has degree equal to the sum of all the exponents appearing on the variables in the term. For example, $-3x^4y^3z^5$ has degree $4 + 3 + 5 = 12$. The degree of a polynomial in more than one variable is equal to the greatest degree of any term appearing in the polynomial. By this definition, the polynomial

$$2x^4y^3 - 3x^5y + x^6y^2 \text{ has degree } 8$$

because of the x^6y^2 term.

A polynomial containing exactly three terms is called a **trinomial;** one containing exactly two terms is a **binomial;** and a single-term polynomial is called a **monomial.** The table shows several examples.

Polynomial	Degree	Type
$9p^7 - 4p^3 + 8p^2$	7	Trinomial
$29x^{11} + 8x^{15}$	15	Binomial
$-10r^6s^8$	14	Monomial
$5a^3b^7 - 3a^5b^5 + 4a^2b^9 - a^{10}$	11	None of these

NOW TRY EXERCISES 33, 35, AND 39. ◀

Addition and Subtraction Since the variables used in polynomials represent real numbers, a polynomial represents a real number. This means that all the properties of the real numbers mentioned in **Section R.2** hold for polynomials. In particular, the distributive property holds, so

$$3m^5 - 7m^5 = (3 - 7)m^5 = -4m^5.$$

Thus, polynomials are added by adding coefficients of like terms; polynomials are subtracted by subtracting coefficients of like terms.

▶ EXAMPLE 4 ADDING AND SUBTRACTING POLYNOMIALS

Add or subtract, as indicated.

(a) $(2y^4 - 3y^2 + y) + (4y^4 + 7y^2 + 6y)$

(b) $(-3m^3 - 8m^2 + 4) - (m^3 + 7m^2 - 3)$

(c) $(8m^4p^5 - 9m^3p^5) + (11m^4p^5 + 15m^3p^5)$

(d) $4(x^2 - 3x + 7) - 5(2x^2 - 8x - 4)$

Solution

(a) $(2y^4 - 3y^2 + y) + (4y^4 + 7y^2 + 6y)$

$= (2 + 4)y^4 + (-3 + 7)y^2 + (1 + 6)y$ Add coefficients of like terms.

$= 6y^4 + 4y^2 + 7y$

(b) $(-3m^3 - 8m^2 + 4) - (m^3 + 7m^2 - 3)$

$= (-3 - 1)m^3 + (-8 - 7)m^2 + [4 - (-3)]$ Subtract coefficients of like terms.

$= -4m^3 - 15m^2 + 7$

(c) $(8m^4p^5 - 9m^3p^5) + (11m^4p^5 + 15m^3p^5) = 19m^4p^5 + 6m^3p^5$

(d) $4(x^2 - 3x + 7) - 5(2x^2 - 8x - 4)$

$$\begin{aligned}
&= 4x^2 - 4(3x) + 4(7) - 5(2x^2) - 5(-8x) - 5(-4) && \text{Distributive property (Section R.2)}\\
&= 4x^2 - 12x + 28 - 10x^2 + 40x + 20 && \text{Multiply.}\\
&= -6x^2 + 28x + 48 && \text{Add like terms.}
\end{aligned}$$

NOW TRY EXERCISES 43 AND 45. ◀

As shown in parts (a), (b), and (d) of Example 4, polynomials in one variable are often written with their terms in **descending order** (or descending degree), so the term of greatest degree is first, the one with the next greatest degree is next, and so on.

Multiplication We also use the associative and distributive properties, together with the properties of exponents, to find the product of two polynomials. To find the product of $3x - 4$ and $2x^2 - 3x + 5$, we treat $3x - 4$ as a single expression and use the distributive property.

$$(3x - 4)(2x^2 - 3x + 5) = (3x - 4)(2x^2) - (3x - 4)(3x) + (3x - 4)(5)$$

Now we use the distributive property three separate times.

$$\begin{aligned}
&= 3x(2x^2) - 4(2x^2) - 3x(3x) - (-4)(3x) + 3x(5) - 4(5)\\
&= 6x^3 - 8x^2 - 9x^2 + 12x + 15x - 20\\
&= 6x^3 - 17x^2 + 27x - 20
\end{aligned}$$

It is sometimes more convenient to write such a product vertically.

Place like terms in the same column.

$$\begin{array}{rrrrl}
 & 2x^2 & -\ 3x & +\ 5 & \\
 & & 3x & -\ 4 & \\ \hline
 & -8x^2 & +\ 12x & -\ 20 & \leftarrow -4(2x^2 - 3x + 5)\\
6x^3 & -\ 9x^2 & +\ 15x & & \leftarrow 3x(2x^2 - 3x + 5)\\ \hline
6x^3 & -\ 17x^2 & +\ 27x & -\ 20 & \text{Add in columns.}
\end{array}$$

▶ EXAMPLE 5 MULTIPLYING POLYNOMIALS

Multiply $(3p^2 - 4p + 1)(p^3 + 2p - 8)$.

Solution

$$\begin{array}{rrrrrrl}
 & & & 3p^2 & -\ 4p & +\ 1 & \\
 & & p^3 & & +\ 2p & -\ 8 & \\ \hline
 & & & -24p^2 & +\ 32p & -\ 8 & \leftarrow -8(3p^2 - 4p + 1)\\
 & & 6p^3 & -\ 8p^2 & +\ 2p & & \leftarrow 2p(3p^2 - 4p + 1)\\
3p^5 & -\ 4p^4 & +\ p^3 & & & & \leftarrow p^3(3p^2 - 4p + 1)\\ \hline
3p^5 & -\ 4p^4 & +\ 7p^3 & -\ 32p^2 & +\ 34p & -\ 8 & \text{Add in columns.}
\end{array}$$

NOW TRY EXERCISE 57. ◀

The **FOIL method** is a convenient way to find the product of two binomials. The memory aid **FOIL** (for **F**irst, **O**utside, **I**nside, **L**ast) gives the pairs of terms to be multiplied to get the product, as shown in the next example.

▶ EXAMPLE 6 USING FOIL TO MULTIPLY TWO BINOMIALS

Find each product.

(a) $(6m + 1)(4m - 3)$ **(b)** $(2x + 7)(2x - 7)$ **(c)** $r^2(3r + 2)(3r - 2)$

Solution

(a)
$$\begin{aligned}(6m + 1)(4m - 3) &= \overset{F}{6m(4m)} + \overset{O}{6m(-3)} + \overset{I}{1(4m)} + \overset{L}{1(-3)}\\ &= 24m^2 - 14m - 3 \qquad -18m + 4m = -14m\end{aligned}$$

(b)
$$\begin{aligned}(2x + 7)(2x - 7) &= 4x^2 - 14x + 14x - 49 \qquad \text{FOIL}\\ &= 4x^2 - 49\end{aligned}$$

(c)
$$\begin{aligned}r^2(3r + 2)(3r - 2) &= r^2(9r^2 - 6r + 6r - 4) \qquad \text{FOIL}\\ &= r^2(9r^2 - 4) \qquad \text{Combine like terms.}\\ &= 9r^4 - 4r^2 \qquad \text{Distributive property}\end{aligned}$$

NOW TRY EXERCISES 49 AND 51. ◀

In part (a) of Example 6, the product of two binomials was a trinomial, while in parts (b) and (c), the product of two binomials was a binomial. The product of two binomials of the forms $x + y$ and $x - y$ is always a binomial. The squares of binomials, $(x + y)^2$ and $(x - y)^2$, are also special products.

SPECIAL PRODUCTS

Product of the Sum and Difference of Two Terms	$(x + y)(x - y) = x^2 - y^2$
Square of a Binomial	$(x + y)^2 = x^2 + 2xy + y^2$ $(x - y)^2 = x^2 - 2xy + y^2$

▶ EXAMPLE 7 USING THE SPECIAL PRODUCTS

Find each product.

(a) $(3p + 11)(3p - 11)$ **(b)** $(5m^3 - 3)(5m^3 + 3)$

(c) $(9k - 11r^3)(9k + 11r^3)$ **(d)** $(2m + 5)^2$

(e) $(3x - 7y^4)^2$

Solution

(a)
$$\begin{aligned}(3p + 11)(3p - 11) &= (3p)^2 - 11^2 \qquad (x + y)(x - y) = x^2 - y^2\\ &= 9p^2 - 121\end{aligned}$$

(b) $(5m^3 - 3)(5m^3 + 3) = (5m^3)^2 - 3^2$

$= 25m^6 - 9$

(c) $(9k - 11r^3)(9k + 11r^3) = (9k)^2 - (11r^3)^2$

$= 81k^2 - 121r^6$

(d) $(2m + 5)^2 = (2m)^2 + 2(2m)(5) + 5^2$ $\quad (x + y)^2 = x^2 + 2xy + y^2$

$= 4m^2 + 20m + 25$

Remember the middle term.

(e) $(3x - 7y^4)^2 = (3x)^2 - 2(3x)(7y^4) + (7y^4)^2$ $\quad (x - y)^2 = x^2 - 2xy + y^2$

$= 9x^2 - 42xy^4 + 49y^8$

NOW TRY EXERCISES 59, 61, 63, AND 65. ◀

▶ **Caution** As shown in Examples 7(d) and (e), ***the square of a binomial has three terms.*** Do not give $x^2 + y^2$ as the result of expanding $(x + y)^2$.

$(x + y)^2 = x^2 + 2xy + y^2$ Remember the middle term.

Also, $(x - y)^2 = x^2 - 2xy + y^2$.

▶ EXAMPLE 8 MULTIPLYING MORE COMPLICATED BINOMIALS

Find each product.

(a) $[(3p - 2) + 5q][(3p - 2) - 5q]$ **(b)** $(x + y)^3$ **(c)** $(2a + b)^4$

Solution

(a) $[(3p - 2) + 5q][(3p - 2) - 5q]$

$= (3p - 2)^2 - (5q)^2$ Product of the sum and difference of terms

$= 9p^2 - 12p + 4 - 25q^2$ Square both quantities.

(b) $(x + y)^3 = (x + y)^2(x + y)$

This does *not* equal $x^3 + y^3$.

$= (x^2 + 2xy + y^2)(x + y)$ Square $x + y$.

$= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$ Multiply.

$= x^3 + 3x^2y + 3xy^2 + y^3$ Combine like terms.

(c) $(2a + b)^4 = (2a + b)^2(2a + b)^2$

$= (4a^2 + 4ab + b^2)(4a^2 + 4ab + b^2)$ Square each $2a + b$.

$= 16a^4 + 16a^3b + 4a^2b^2 + 16a^3b + 16a^2b^2$

$+ 4ab^3 + 4a^2b^2 + 4ab^3 + b^4$

$= 16a^4 + 32a^3b + 24a^2b^2 + 8ab^3 + b^4$

NOW TRY EXERCISES 69, 73, AND 75. ◀

Division The quotient of two polynomials can be found with an algorithm (that is, a step-by-step procedure [or "recipe"]) for long division similar to that used for dividing whole numbers. ***Both polynomials must be written in descending order.***

▶ EXAMPLE 9 DIVIDING POLYNOMIALS

Divide $4m^3 - 8m^2 + 4m + 6$ by $2m - 1$.

Solution

$4m^3$ divided by $2m$ is $2m^2$.

$-6m^2$ divided by $2m$ is $-3m$.

m divided by $2m$ is $\frac{1}{2}$.

$$\begin{array}{rrl}
 & 2m^2 - 3m + \frac{1}{2} & \\
2m - 1\,\big) & \overline{4m^3 - 8m^2 + 4m + 6} & \\
 & \underline{4m^3 - 2m^2} & \longleftarrow 2m^2(2m - 1) = 4m^3 - 2m^2 \\
 & -6m^2 + 4m & \longleftarrow \text{Subtract; bring down the next term.} \\
 & \underline{-6m^2 + 3m} & \longleftarrow -3m(2m - 1) = -6m^2 + 3m \\
 & m + 6 & \longleftarrow \text{Subtract; bring down the next term.} \\
 & \underline{m - \frac{1}{2}} & \longleftarrow \frac{1}{2}(2m - 1) = m - \frac{1}{2} \\
 & \frac{13}{2} & \longleftarrow \text{Subtract; the remainder is } \frac{13}{2}.
\end{array}$$

To subtract, add the opposite.

$$\text{Thus, } \quad \frac{4m^3 - 8m^2 + 4m + 6}{2m - 1} = 2m^2 - 3m + \frac{1}{2} + \frac{\frac{13}{2}}{2m - 1}.$$

Remember to add $\frac{\text{remainder}}{\text{divisor}}$.

In the first step of this division, $4m^3 - 2m^2$ is subtracted from $4m^3 - 8m^2 + 4m + 6$. The complete result, $-6m^2 + 4m + 6$, should be written under the line. However, to save work we "bring down" just the $4m$, the only term needed for the next step.

NOW TRY EXERCISE 91. ◀

The polynomial $3x^3 - 2x^2 - 150$ has a missing term, the term in which the power of x is 1. When a polynomial has a missing term, we allow for that term by inserting a term with a 0 coefficient for it.

▶ EXAMPLE 10 DIVIDING POLYNOMIALS WITH MISSING TERMS

Divide $3x^3 - 2x^2 - 150$ by $x^2 - 4$.

Solution Both polynomials have missing first-degree terms. Insert each missing term with a 0 coefficient.

$$\begin{array}{rrl}
 & 3x - 2 \qquad\qquad & \\
x^2 + \underset{\text{Missing term}}{0x} - 4\,\big) & \overline{3x^3 - 2x^2 + \overset{\text{Missing term}}{0x} - 150} & \longleftarrow \text{Insert placeholders for missing terms.} \\
 & \underline{3x^3 + 0x^2 - 12x} \qquad & \\
 & -2x^2 + 12x - 150 & \\
 & \underline{-2x^2 + 0x + 8} & \\
 & 12x - 158 & \longleftarrow \text{Remainder}
\end{array}$$

The division process ends when the remainder is 0 or the degree of the remainder is less than that of the divisor. Since $12x - 158$ has lesser degree than the divisor $x^2 - 4$, it is the remainder. Thus,

$$\frac{3x^3 - 2x^2 - 150}{x^2 - 4} = 3x - 2 + \frac{12x - 158}{x^2 - 4}.$$

NOW TRY EXERCISE 93. ◀

R.3 Exercises

Concept Check Decide whether each expression has been simplified correctly. If not, correct it. Assume all variables represent nonzero real numbers.

1. $(mn)^2 = mn^2$ **2.** $y^2 \cdot y^5 = y^7$ **3.** $\left(\frac{k}{5}\right)^3 = \frac{k^3}{5}$ **4.** $3^0y = 0$

5. $4^5 \cdot 4^2 = 16^7$ **6.** $(a^2)^3 = a^5$ **7.** $cd^0 = 1$ **8.** $(2b)^4 = 8b^4$

9. $\left(\frac{1}{4}\right)^5 = \frac{1}{4^5}$ **10.** $x^3 \cdot x \cdot x^4 = x^8$

Simplify each expression. See Example 1.

11. $9^3 \cdot 9^5$ **12.** $4^2 \cdot 4^8$ **13.** $(-4x^5)(4x^2)$

14. $(3y^4)(-6y^3)$ **15.** $n^6 \cdot n^4 \cdot n$ **16.** $a^8 \cdot a^5 \cdot a$

17. $(-3m^4)(6m^2)(-4m^5)$ **18.** $(-8t^3)(2t^6)(-5t^4)$

Simplify each expression. See Examples 1–3.

19. $(2^2)^5$ **20.** $(6^4)^3$ **21.** $(-6x^2)^3$ **22.** $(-2x^5)^5$

23. $-(4m^3n^0)^2$ **24.** $(2x^0y^4)^3$ **25.** $\left(\frac{r^8}{s^2}\right)^3$ **26.** $-\left(\frac{p^4}{q}\right)^2$

27. $\left(\frac{-4m^2}{t}\right)^4$ **28.** $\left(\frac{-5n^4}{r^2}\right)^3$

Match each expression in Column I with its equivalent in Column II. See Example 3.

	I	II		I	II
29.	**(a)** 6^0	**A.** 0	**30.**	**(a)** $3p^0$	**A.** 0
	(b) -6^0	**B.** 1		**(b)** $-3p^0$	**B.** 1
	(c) $(-6)^0$	**C.** −1		**(c)** $(3p)^0$	**C.** −1
	(d) $-(-6)^0$	**D.** 6		**(d)** $(-3p)^0$	**D.** 3
		E. −6			**E.** −3

31. Explain why $x^2 + x^2 \neq x^4$. **32.** Explain why $(x + y)^2 \neq x^2 + y^2$.

Concept Check Identify each expression as a polynomial *or* not a polynomial. *For each polynomial, give the degree and identify it as a* monomial, binomial, trinomial, *or* none of these.

33. $-5x^{11}$ **34.** $9y^{12} + y^2$ **35.** $18p^5q + 6pq$

36. $2a^6 + 5a^2 + 4a$ **37.** $\sqrt{2}x^2 + \sqrt{3}x^6$ **38.** $-\sqrt{7}m^5n^2 + 2\sqrt{3}m^3n^2$

39. $\frac{1}{3}r^2s^2 - \frac{3}{5}r^4s^2 + rs^3$ **40.** $\frac{13}{10}p^7 - \frac{2}{7}p^5$ **41.** $\frac{5}{p} + \frac{2}{p^2} + \frac{5}{p^3}$

42. $-5\sqrt{z} + 2\sqrt{z^3} - 5\sqrt{z^5}$

Find each sum or difference. See Example 4.

43. $(5x^2 - 4x + 7) + (-4x^2 + 3x - 5)$

44. $(3m^3 - 3m^2 + 4) + (-2m^3 - m^2 + 6)$

45. $2(12y^2 - 8y + 6) - 4(3y^2 - 4y + 2)$

46. $3(8p^2 - 5p) - 5(3p^2 - 2p + 4)$

47. $(6m^4 - 3m^2 + m) - (2m^3 + 5m^2 + 4m) + (m^2 - m)$

48. $-(8x^3 + x - 3) + (2x^3 + x^2) - (4x^2 + 3x - 1)$

Find each product. See Examples 5 and 6.

49. $(4r - 1)(7r + 2)$ **50.** $(5m - 6)(3m + 4)$

51. $x^2\left(3x - \frac{2}{3}\right)\left(5x + \frac{1}{3}\right)$ **52.** $\left(2m - \frac{1}{4}\right)\left(3m + \frac{1}{2}\right)$

53. $4x^2(3x^3 + 2x^2 - 5x + 1)$ **54.** $2b^3(b^2 - 4b + 3)$

55. $(2z - 1)(-z^2 + 3z - 4)$ **56.** $(k + 2)(12k^3 - 3k^2 + k + 1)$

57. $(m - n + k)(m + 2n - 3k)$ **58.** $(r - 3s + t)(2r - s + t)$

Find each product. See Examples 7 and 8.

59. $(2m + 3)(2m - 3)$ **60.** $(8s - 3t)(8s + 3t)$ **61.** $(4x^2 - 5y)(4x^2 + 5y)$

62. $(2m^3 + n)(2m^3 - n)$ **63.** $(4m + 2n)^2$ **64.** $(a - 6b)^2$

65. $(5r - 3t^2)^2$ **66.** $(2z^4 - 3y)^2$

67. $[(2p - 3) + q]^2$ **68.** $[(4y - 1) + z]^2$

69. $[(3q + 5) - p][(3q + 5) + p]$ **70.** $[(9r - s) + 2][(9r - s) - 2]$

71. $[(3a + b) - 1]^2$ **72.** $[(2m + 7) - n]^2$

73. $(y + 2)^3$ **74.** $(z - 3)^3$

75. $(q - 2)^4$ **76.** $(r + 3)^4$

Perform the indicated operations. See Examples 4–8.

77. $(p^3 - 4p^2 + p) - (3p^2 + 2p + 7)$ **78.** $(2z + y)(3z - 4y)$

79. $(7m + 2n)(7m - 2n)$ **80.** $(3p + 5)^2$

81. $-3(4q^2 - 3q + 2) + 2(-q^2 + q - 4)$ **82.** $2(3r^2 + 4r + 2) - 3(-r^2 + 4r - 5)$

83. $p(4p - 6) + 2(3p - 8)$ **84.** $m(5m - 2) + 9(5 - m)$

85. $-y(y^2 - 4) + 6y^2(2y - 3)$ **86.** $-z^3(9 - z) + 4z(2 + 3z)$

Perform each division. See Examples 9 and 10.

You may use synthetic Division or LONG DIV.

87. $\frac{-4x^7 - 14x^6 + 10x^4 - 14x^2}{-2x^2}$ **88.** $\frac{-8r^3s - 12r^2s^2 + 20rs^3}{4rs}$

89. $\frac{10x^8 - 16x^6 - 4x^4}{-2x^6}$ **90.** $\frac{3x^3 - 2x + 5}{x - 3}$

91. $\frac{6m^3 + 7m^2 - 4m + 2}{3m + 2}$ **92.** $\frac{6x^4 + 9x^3 + 2x^2 - 8x + 7}{3x^2 - 2}$